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High Field Science with Lasers

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FZU Institute of Physics of the Czech Academy of Sciences

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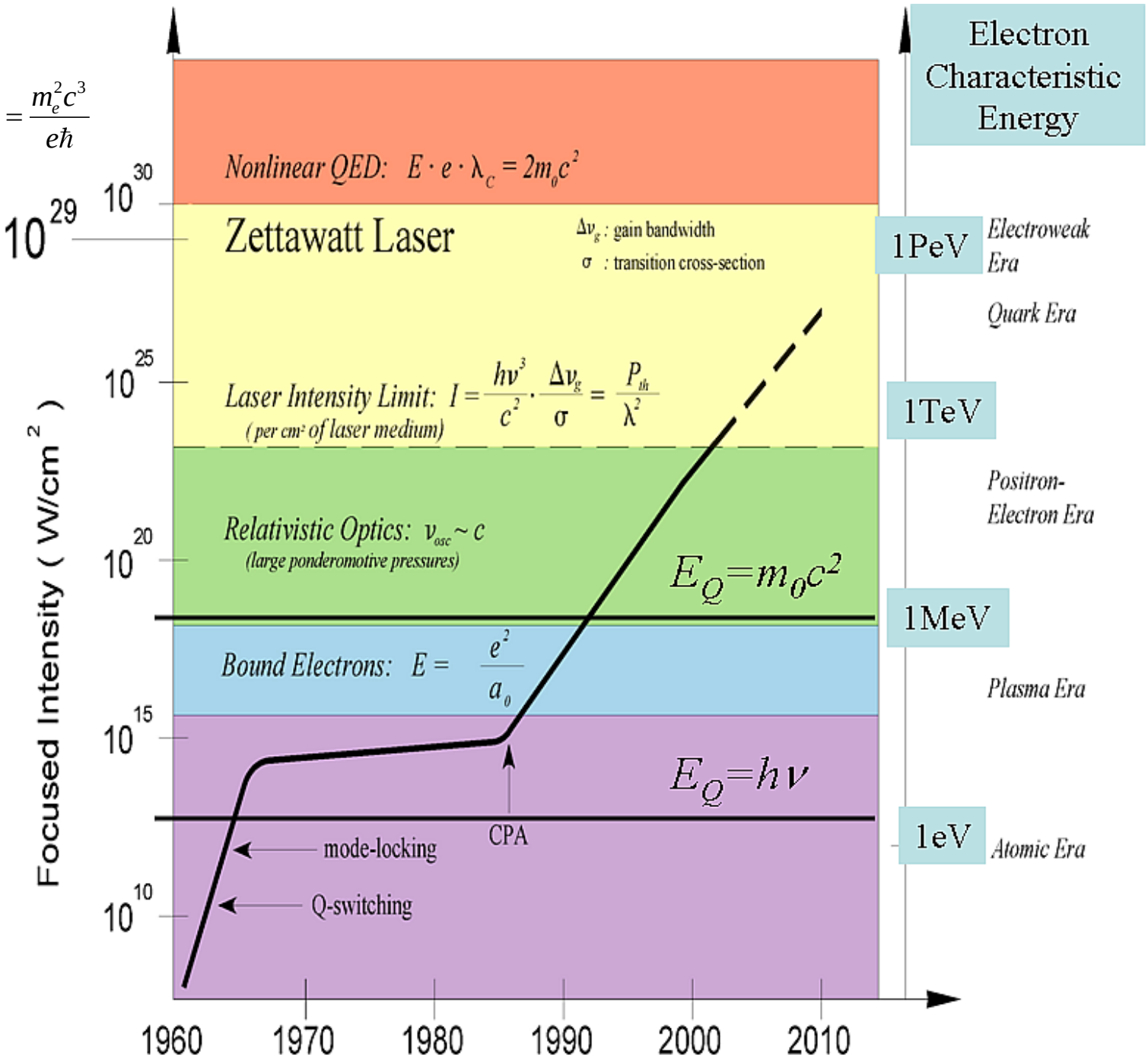
$$E_S = \frac{m_e c^2}{e\lambda\!\!\!^{-}_C} = \frac{m_e^2 c^3}{e\hbar}$$

$$I = c\frac{E^2}{4\pi}$$



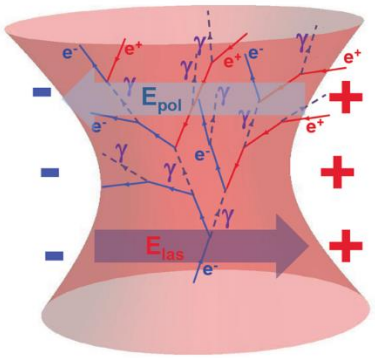
G. A. Mourou, T. Tajima, and S. V. Bulanov, Rev. Mod. Phys. 78, 309 (2006)

Extreme Field Limits: Nonlinear QED Vacuum

Towards studying of nonlinear QED effects with high power lasers.
Radiation dominated & QED regimes in the high intensity electromagnetic wave interaction with charged particles & vacuum.

$$E_S = \frac{m_e^2 c^3}{e\hbar} \qquad I_S = c\frac{E_S^2}{4\pi} \approx 10^{29}\,\frac{W}{cm^2}$$

Schwinger (Sauter, Bohr,…) field



Extreme field limits

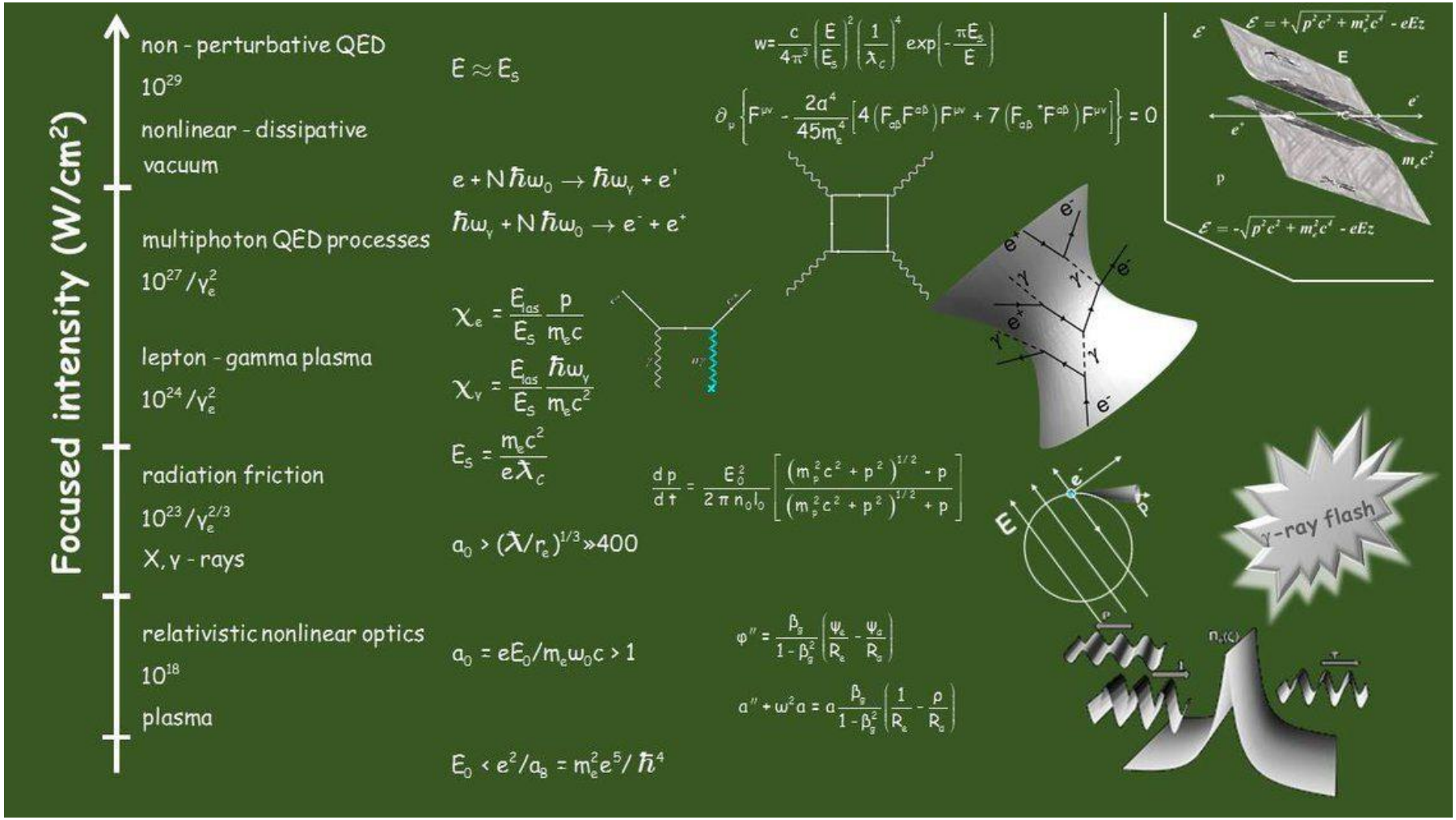
The critical electric field of quantum electrodynamics, called also the Schwinger field, is so strong that it produces electron-positron pairs from vacuum, converting the energy of light into matter. Since the dawn of quantum electrodynamics there has been a dream on how to reach it on Earth. With the rise of the laser technologies this field becomes feasible through construction of extreme power lasers or/and with the sophisticated usage of nonlinear processes in relativistic plasmas. This is one of the most attractive motivations for extreme power laser development, i.e. producing matter from vacuum by pure light in fundamental process of quantum electrodynamics in the non-perturbative regime.

The laser with intensity well below the Schwinger limit can create an avalanche of electron-positron pairs similar to a discharge before attaining the Schwinger field. It was also realized that the Schwinger limit can be reached using an appropriate configuration of laser beams.

The regimes of dominant radiation reaction, which completely changes the electromagnetic wave-matter interaction, will be revealed. This will result in a new powerful source of high brightness gamma-rays. A possibility of the demonstration of the electron-positron pair creation in vacuum in a multi-photon processes can be realized. This will allow modeling under terrestrial laboratory conditions the processes in various astrophysical objects.

An ultra-relativistic electron emits Cherenkov radiation in vacuum with an induced by strong electromagnetic wave refraction index larger than unity. During the interaction with this wave the electron also radiates photons via the Compton scattering. Synergic Cherenkov-Compton process can be observed by colliding laser accelerated electrons with a high intensity electromagnetic pulse.

Extreme Field Limits



https://www.eli-beams.eu/projekty/hifi/

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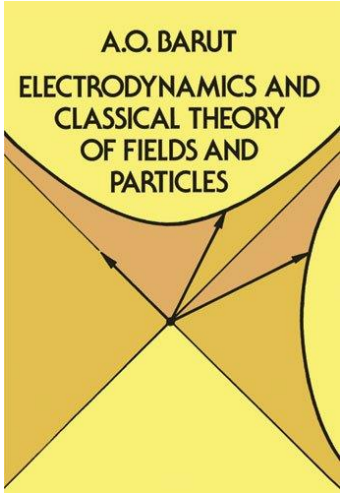
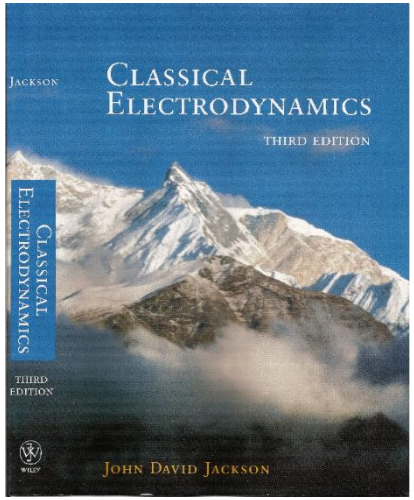

A.O. BARUT
ELECTRODYNAMICS AND CLASSICAL THEORY OF FIELDS AND PARTICLES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CLASSICAL ELECTRODYNAMICS
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CLASSICAL ELECTRODYNAMICS
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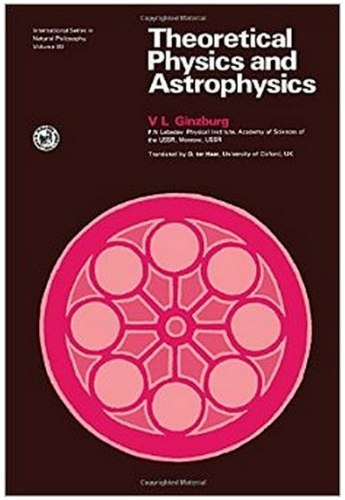
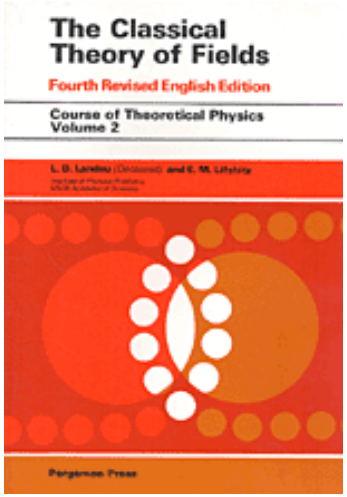
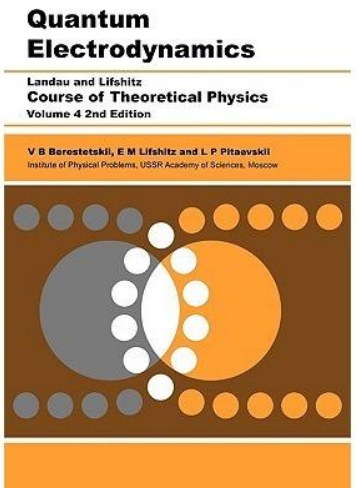
WILEY
JOHN DAVID JACKSON


Theoretical Physics and Astrophysics
V L Ginzburg


The Classical Theory of Fields
Fourth Revised English Edition
Course of Theoretical Physics
Volume 2
L. D. Landau (Deceased) and E. M. Lifshitz
Pergamon Press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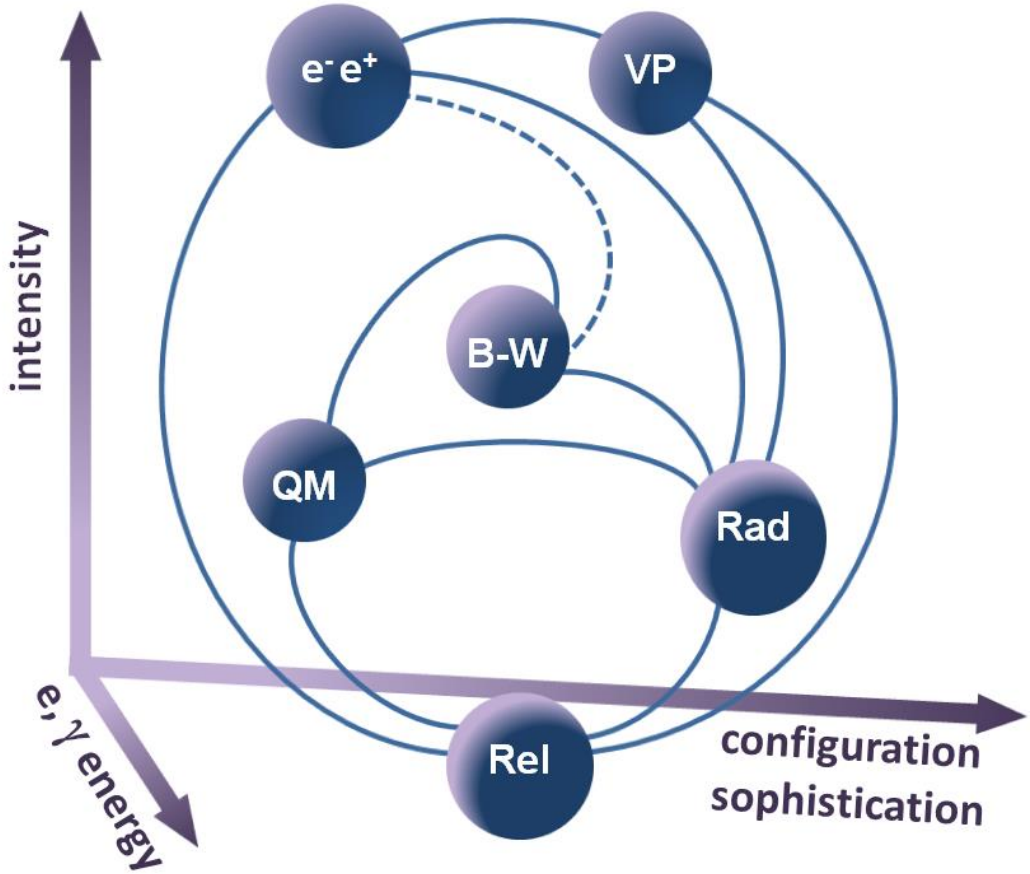
Schematic of nonlinear QED processes



Laser-matter interaction transition from the relativistic regime to dominant radiation friction and nonlinear Thomson scattering through the limit when quantum electrodynamics comes into play and to electron-positron avalanche/cascade development, towards nonlinear vacuum with electron-positron creation and nonlinear vacuum polarization while the EM field intensity increases.

Dimensionless parameters

1. Normalized EM wave amplitude

$$a = \frac{e\sqrt{\left(A_\mu\right)^2}}{m_e \omega c} = \frac{eE\lambda}{2\pi m_e c^2} \quad \overset{a=1}{\Leftrightarrow} \quad I_{rel} = \frac{m_e^2 c^3 \omega^2 a^2}{4\pi e^2} = 1.37\times 10^{18}\left(\frac{1\mu m}{\lambda}\right)^2 \frac{W}{cm^2}$$

2. Radiation friction becomes important for

$$a > a_{rad} = \varepsilon_{rad}^{-1/3} \text{ with } \varepsilon_{rad} = \frac{4\pi r_e}{3\lambda} \quad \overset{a=a_{rad}}{\Leftrightarrow} \quad I_R = \frac{3m_e^{8/3} c^5 \omega^{4/3}}{8\pi e^{10/3}} = 2.65\times 10^{23}\left(\frac{1\mu m}{\lambda}\right)^{4/3} \frac{W}{cm^2}$$

where $r_e = e^2 / m_e c^2$ - classical electron radius

3. QED parameters

$$\chi_e = \frac{e\hbar\sqrt{\left(F^{\mu\nu} p_\mu\right)^2}}{m_e^3 c^4} \approx \frac{E}{E_S}\frac{p_e}{m_e c} \quad \text{and} \quad \chi_\gamma = \frac{e\hbar^2\sqrt{\left(F^{\mu\nu} k_\mu\right)^2}}{m_e^3 c^4} \approx a\frac{\hbar^2 \omega_0 \omega_\gamma}{m_e^2 c^4} \quad \left[N\omega_0 + \omega_\gamma \to e^+ e^-\right]$$

4. QED limit :

$$\chi_e \approx 1 \qquad I_Q = \frac{m_e^3 c^5 \omega}{8\pi e^2 \hbar} = 5.75\times 10^{23}\left(\frac{1\mu m}{\lambda}\right)\frac{W}{cm^2}$$

$$E_S = \frac{m_e^2 c^3}{e\hbar} \quad \Leftrightarrow \quad a_S = \frac{m_e c^2}{\hbar\omega} \quad \overset{a=a_S}{\Leftrightarrow} \quad I_S = \frac{m_e^4 c^7}{4\pi e^2 \hbar^2} = 2.36\times 10^{29}\frac{W}{cm^2}$$

H. R. Reiss, J. Math. Phys. (N.Y.) 3, 59 (1962)
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R. Ruffini, G. Vereshchagin, S. S. Xue, Physics Reports 487, 1 (2010)
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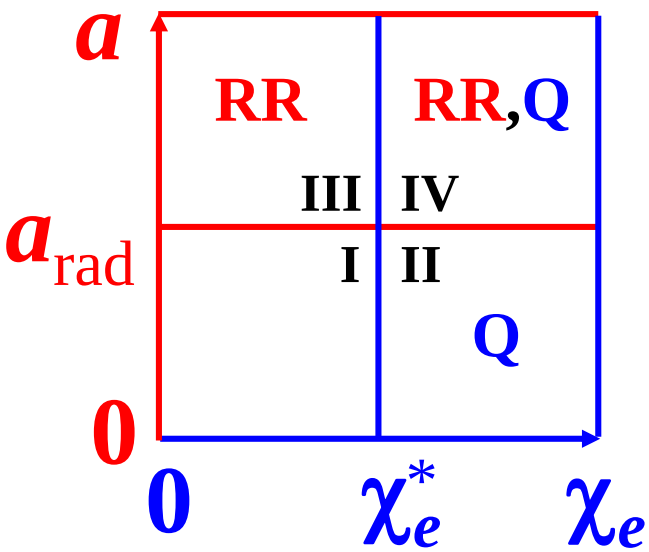
Electron Motion in High Intensity EM Wave

4 regimes



Dimensionless amplitude

$$a = eE/m_e\omega c$$

At $a=a_{rad}$ emitted energy becomes equal to the energy received from EM wave.

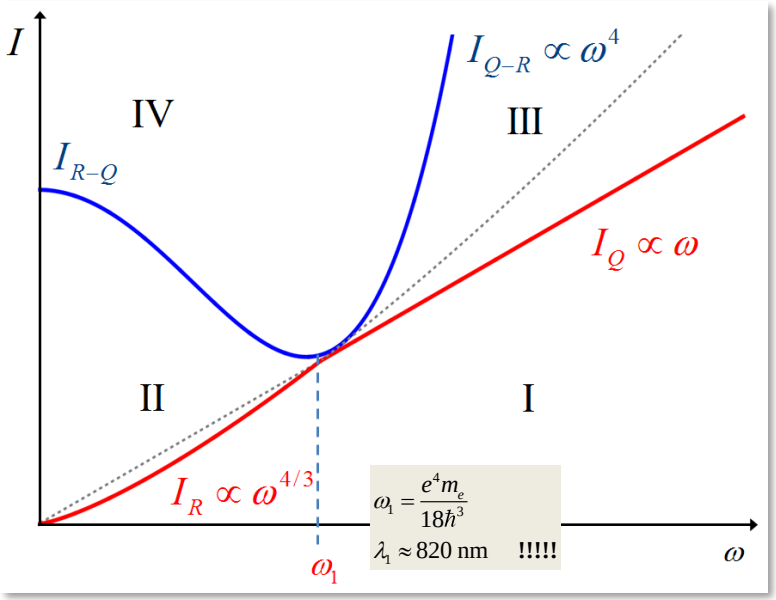
$$a_{rad} = \left(\frac{3\lambda}{4\pi r_e}\right)^{1/3} \qquad r_e = \frac{e^2}{m_e c^2}$$

When the recoil of the emitted photon is significant, the emission probability is characterized by χ_e parameter (Lorentz and gauge invariant)

$$\chi_e = (\gamma_e/E_S)[(\mathbf{E} + \boldsymbol{\beta} \times \mathbf{B})^2 - (\boldsymbol{\beta} \cdot \mathbf{E})^2]^{1/2}$$

At $\chi_e > \chi_e^* \approx 1$ the QED effects come into play

Four Interaction Domains



$$I_R = \frac{m_e^4 c^5 e^2}{144\pi\hbar^4}\left(\frac{\omega}{\omega_1}\right)^{4/3} = 3.8\times10^{23}\left(\frac{\omega}{\omega_1}\right)^{4/3}\frac{\mathrm{W}}{\mathrm{cm}^2}$$

$$I_Q = \frac{m_e^4 c^5 e^2}{144\pi\hbar^4}\left(\frac{\omega}{\omega_1}\right) = 3.8\times10^{23}\left(\frac{\omega}{\omega_1}\right)\frac{\mathrm{W}}{\mathrm{cm}^2}$$

$$I_{R-Q} = \frac{m_e^4 c^5 e^2}{9\pi\hbar^4} = 5.6\times10^{24}\,\frac{\mathrm{W}}{\mathrm{cm}^2}$$

$$I_{Q-R} = 87\,\frac{c^5\hbar^8\omega^4}{e^{14}} = 8.2\times10^{21}\left(\frac{\omega}{\omega_1}\right)^4\frac{\mathrm{W}}{\mathrm{cm}^2}$$

SVB, T. Zh. Esirkepov, M. Kando, J. Koga, K. Kondo, and G. Korn,
***Plasma Phys. Rep.* 41, 1-51 (2015)**

Curves $I_R(w)$, $I_Q(w)$ and $I_{R-Q}(w)$, $I_{Q-R}(w)$ subdivide (I, w) plane to 4 domains:

I) Relativistic electron - EM field interaction with neither radiation friction nor QED effects

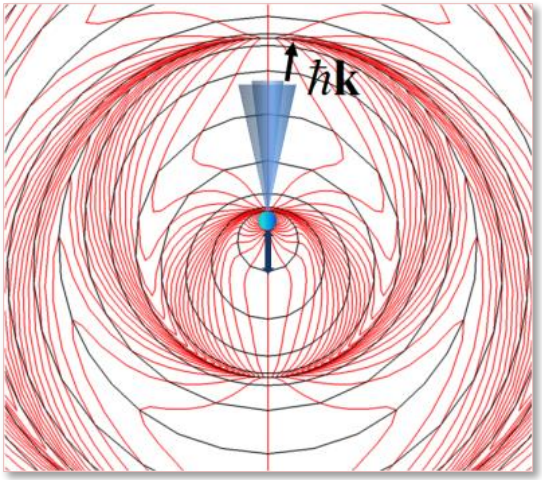
II) Electron - EM wave interaction is dominated by radiation friction

III) QED effects important with insignificant radiation friction effects

IV) Both QED and radiation friction determine radiating charged particle dynamics in the EM field

Radiation Friction Force:

Lorentz-Abraham-Dirac, Pomeranchuk, Landau-Lifshitz, …



Basic Equations: Minkovski & Maxwell Equations

Minkovski equations: $\dfrac{du^\mu}{ds} = \dfrac{e}{m_e c} F^\mu{}_\nu u^\nu$ **where** $u^\mu = \dfrac{dx^\mu}{ds}, \quad ds = \dfrac{dt}{\gamma}$

EM field tensor: $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

Maxwell equations: $\varepsilon^{\mu\nu\sigma\rho} \partial_\rho F_{\nu\sigma} = 0$ **and** $\partial_\nu F^{\mu\nu} = \dfrac{4\pi}{c} j^\mu$

In 3D notations: $\dot{\boldsymbol{p}} = e\left(\boldsymbol{E} + \dfrac{1}{c} \boldsymbol{v} \times \boldsymbol{B} \right), \quad \dot{\boldsymbol{x}} = c \dfrac{\boldsymbol{p}}{m_e \gamma}, \quad \gamma = \left(1 + \dfrac{p_\mu p^\nu}{m_e^2 c^2} \right)^{1/2}$

$$\nabla \times \boldsymbol{B} = \frac{4\pi}{c} \boldsymbol{j} + \frac{1}{c} \partial_t \boldsymbol{E}$$

$$\nabla \cdot \boldsymbol{B} = 0$$

$$\nabla \times \boldsymbol{E} = -\frac{1}{c} \partial_t \boldsymbol{B}$$

$$\nabla \cdot \boldsymbol{E} = 4\pi\rho$$

Radiation Losses

Intensity of radiation emitted by electron is given by

$$I = \frac{2e^2}{3m_e^2c^3}\left(\frac{dp_i}{ds}\frac{dp_i}{ds}\right)$$

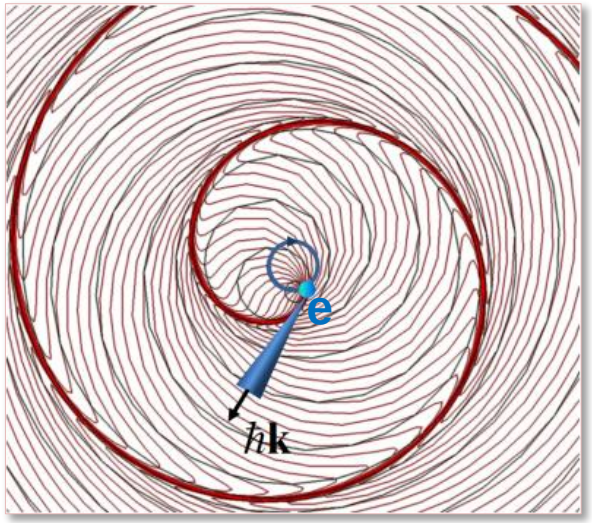
In <u>circularly polarized</u> EM wave (**in plasma**), whose

amplitude is equal to $a_0 = \dfrac{eE_0}{m_e\omega_0 c}$ electron energy

losses are $$\dot{\mathcal{E}}^{(-)} = \frac{2e^4E_0^2}{3m_e^2c^3}\left[1+\left(\frac{eE_0}{m_e\omega_0 c}\right)^2\right]$$

For <u>linearly polarized</u> wave we have

$$\dot{\mathcal{E}}^{(-)} = \frac{e^4E_0^2}{3m_e^2c^3}\left[1+\frac{3}{8}\left(\frac{eE_0}{m_e\omega_0 c}\right)^2\right]$$



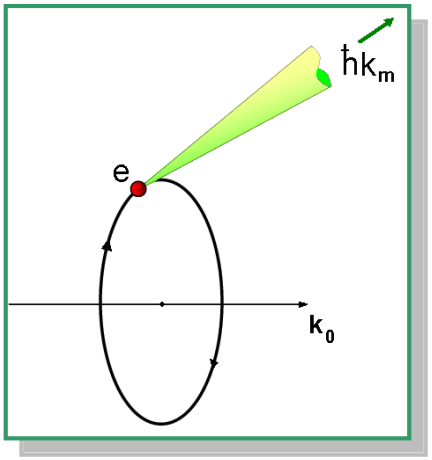
Pattern of field emitted by electron. T.Shintake, 2003

L.D.Landau & E.M.Lifshitz
"The Classical Theory of Fields"

LAD-form of radiation friction force

Equations of electron motion are:

$$m_e c^2 \frac{du^\mu}{ds} = \frac{e}{c} F^{\mu\nu} u_\nu + g^\mu$$



Radiation friction force is given by

$$g^\mu = \frac{2e^2}{3c}\left(\frac{d^2u^\mu}{ds^2} - u^\mu u^\nu \frac{d^2u_\nu}{ds^2}\right)$$

Here $\mu = 0,1,2,3$, s is proper time: $ds = c\,dt/\gamma$

4-velocity is $u^i = \frac{dx^i}{ds} = \left(\gamma, \frac{\boldsymbol{p}}{m_e c}\right)$

and $F_{\mu\nu} = \frac{\partial A_\nu}{\partial x^\mu} - \frac{\partial A_\mu}{\partial x^\nu}$ is 4-tensor of EM field

Covariant and 3D forms of L-L expression

Radiation reaction force in the Landau-Lifshitz approximation (L-L II, §76):

$$m_e c^2 \frac{du^\mu}{ds} = \frac{e}{c} F^{\mu\nu} u_\nu + \frac{2e^3}{3m_e c^3}\left\{ \partial_\lambda F^{\mu\nu} u_\nu u_\lambda - \frac{e}{m_e c^2}\left[F^{\mu\lambda} F_{\nu\lambda} u^\nu - \left(F_{\nu\lambda} u^\lambda\right)\left(F^{\nu\kappa} u_\kappa\right) u^\mu \right]\right\}$$

$$m_e c^2 \frac{d\mathbf{p}}{dt} = e\left(\mathbf{E} + \frac{\mathbf{v}}{c}\times\mathbf{B}\right) +$$

$$+\frac{2e^3}{3m_e c^3}\gamma\left\{\left(\partial_t + \mathbf{v}\cdot\nabla\right)\mathbf{E} + \frac{1}{c}\mathbf{v}\times\left(\left(\partial_t + \mathbf{v}\cdot\nabla\right)\mathbf{B}\right)\right\} + \qquad \propto \varepsilon_{rad}\,\gamma\,\omega\,a_0$$

$$+\frac{2e^4}{3m_e^2 c^4}\left\{\mathbf{E}\times\mathbf{B} + \frac{1}{c}\mathbf{B}\times\left(\mathbf{B}\times\mathbf{v}\right) + \frac{1}{c}\mathbf{E}\left(\mathbf{v}\cdot\mathbf{E}\right)\right\} - \qquad \propto \varepsilon_{rad}\,a_0^2$$

$$-\frac{2e^4}{3m_e^2 c^5}\gamma^2\mathbf{v}\left\{\left(\mathbf{E} + \frac{\mathbf{v}}{c}\times\mathbf{B}\right)^2 - \frac{1}{c^2}\left(\mathbf{v}\cdot\mathbf{E}\right)^2\right\} \qquad \propto \varepsilon_{rad}\,\gamma^2\,a_0^2$$

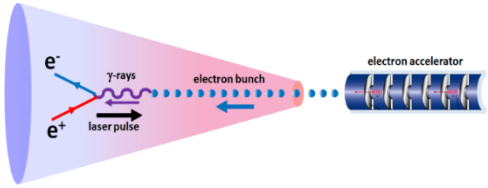
Dimensionless parameter:

$$\varepsilon_{rad} = \frac{2e^2\omega}{3m_e c^3} = \frac{4\pi r_e}{3\lambda} \approx 10^{-8}\left(\frac{1\mu m}{\lambda}\right)$$

Weak field approximation:

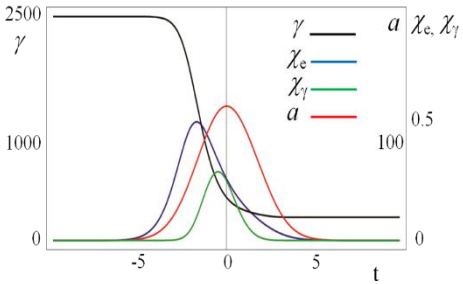
$$E < E_{cl} = \frac{e}{r_e^2} = \frac{m_e^2 c^4}{e^3} = \frac{1}{\alpha}E_S \qquad \alpha = \frac{e^2}{\hbar c} \approx \frac{1}{137}$$

“Pomeranchuk Theorem”



We consider the electron colliding with the EM wave given by

$$a_0(t - x/c) \underset{x=-ct}{\approx} a_0(2t)$$



Retaining the main order terms in the L-L radiation friction force, we obtain equation for the *x*-component of the electron momentum

$$\frac{d\gamma}{dt} = -\varepsilon_{rad}\omega a_0^2(2t)\gamma^2, \qquad \varepsilon_{rad} = \frac{2r_e}{3\lambda\!\!\!^{-}}$$

Its solution is

$$\gamma(t) = \frac{\gamma_0}{1 + \varepsilon_{rad}\omega\gamma_0\int_0^t a_0^2(2t')dt'}, \quad \gamma \underset{t\to\infty}{\longrightarrow} \frac{1}{\varepsilon_{rad}\omega\tau_{las}a_0^2}$$

For $\varepsilon_{rad} = 10^{-8}$ $\omega\tau_{las} = 100$ $a_0 = 300$ the electron gamma - factor becomes equal to

$$\gamma = 10$$

Landau, L. D. and Lifshitz, E. M., *The Classical Theory of Fields, Pergamon*, 1975

Pomeranchuk, I.: ‘Maximum Energy that Primary Cosmic-ray Electrons Can Acquire on the Surface of the Earth as a Result of Radiation in the Earth's Magnetic Field’, J. Phys. USSR, 2, 65 – 69, 1940

Radiation friction effects on charged particle motion

Equations of electron motion

$$m_e c^2 \frac{du^\mu}{ds} = \frac{e}{c} F^{\mu\nu} u_\nu + g^\mu$$

with radiation friction force g^μ

EM wave is modelled by rotating E field

$$\dot{\mathbf{q}} = -\boldsymbol{a} - \frac{\varepsilon_{\text{rad}} G_e(\chi_e)}{\gamma_e} \left\{ \gamma^2 \dot{\boldsymbol{a}} - \boldsymbol{a}(\mathbf{q}\cdot\boldsymbol{a}) + \boldsymbol{q}\left[(\gamma\boldsymbol{a})^2 - (\mathbf{q}\cdot\boldsymbol{a})^2 \right] \right\}$$

$$n = \frac{n}{n_{cr}}, \quad \tau = \Omega t, \quad \mathbf{q} = \frac{\mathbf{p}}{m_e c}, \quad \boldsymbol{a} = \frac{e\mathbf{E}}{m_e \Omega c}, \quad \gamma_e = \left(1 + q_1^2 + q_2^2 + q_3^2\right)^{1/2}$$

QED effects incorporated with the form - factor, $G_e(\chi_e)$, equal to the ratio of the full radiation intensity to the intensity emitted by a classical electron

$$G_e(\chi_e) = -\frac{3}{4} \int_0^\infty \left[\frac{4 + \chi_e x^{3/2} + 4\chi_e^2 x^3}{\left(1 + \chi_e x^{3/2}\right)^4} \right] \Phi'(x) x dx$$

where $\Phi(x)$ is the Airy function

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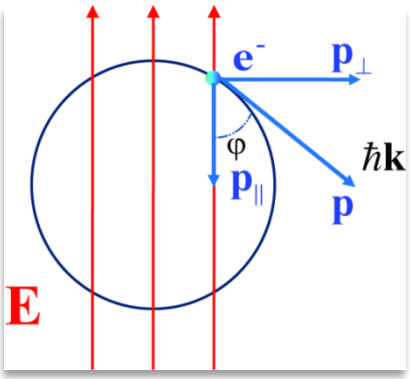
A. Di Piazza, et al., Rev. Mod. Phys. 84, 1177 (2012)

Radiation friction effect on electron dynamics

In order to describe the electron motion we write the electron momentum as

$$\begin{pmatrix} q_1 \\ q_{\|} \\ q_{\perp} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\tau) & \sin(\tau) \\ 0 & -\sin(\tau) & \cos(\tau) \end{pmatrix} \begin{pmatrix} q_1 \\ q_2 \\ q_3 \end{pmatrix}$$



Here $q_{\|}$ and $q_{\perp}$ are the components of the electron momentum parallel and perpendicular to the electric field.

We assume here that the wave is given.

Neglecting the change of the q_1 –component (near-critical plasma density), we obtain

$$\dot{q}_{\perp} - q_{\|} = -\varepsilon_{\text{rad}} G_e(\chi_e)\left[\gamma_e a + a^2 \frac{q_{\perp}}{\gamma_e}\left(1 + q_{\perp}^2\right)\right]$$

$$\dot{q}_{\|} + q_{\perp} = a - \varepsilon_{\text{rad}} G_e(\chi_e) a^2 q_{\|} \frac{q_{\perp}^2}{\gamma_e}$$

with the energy balance equation

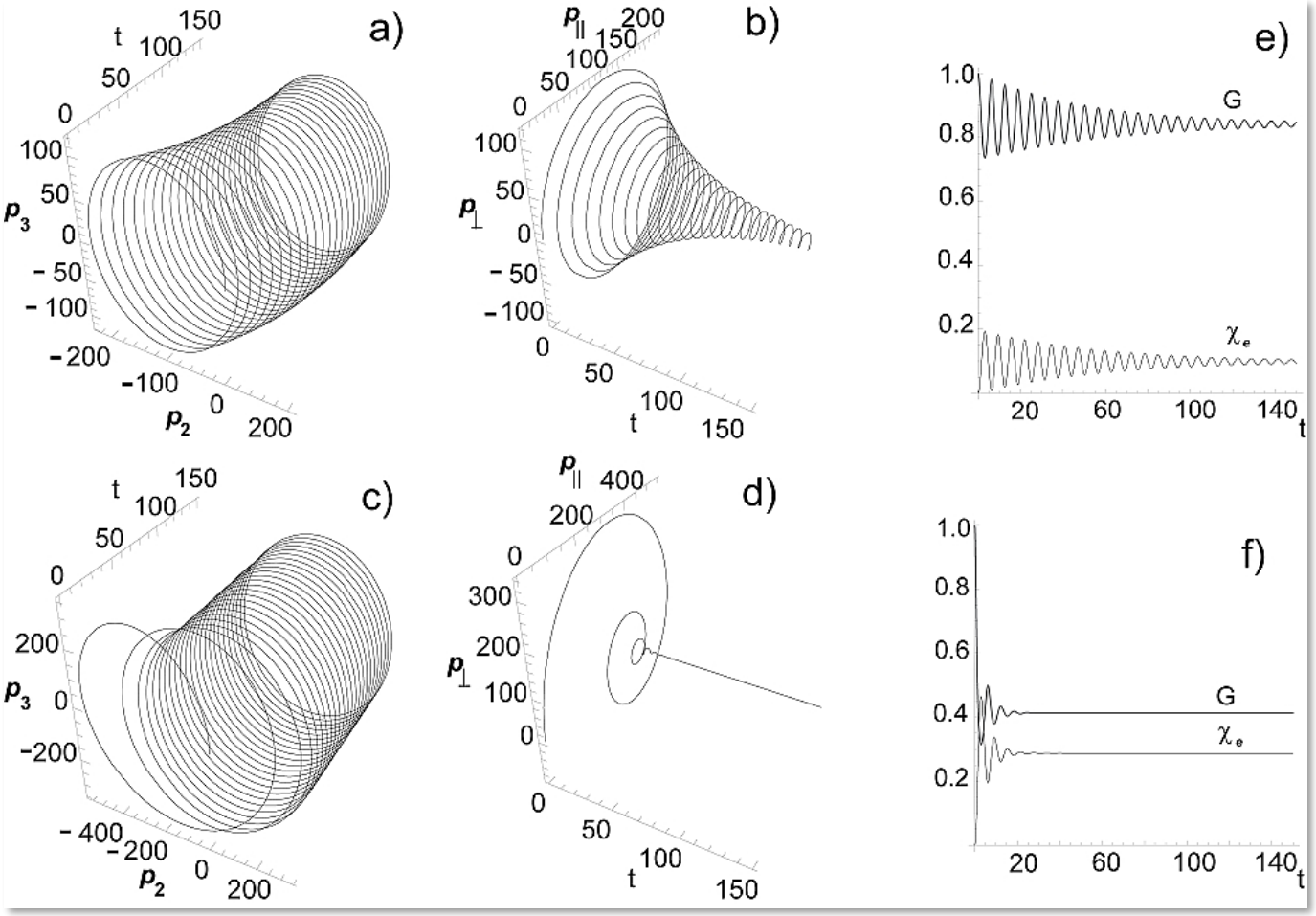
$$\dot{\gamma}_e = a\, u_{\|} - \varepsilon_{\text{rad}} G_e(\chi_e)\left(a q_{\perp} + a^2 q_{\perp}^2\right)$$

The QED parameter is given by

$$\chi_e = \frac{a}{a_S}\sqrt{1 + q_{\perp}^2}$$

S. V. Bulanov, T. Zh. Esirkepov, M. Kando, J. K. Koga, S. S. Bulanov, "Lorentz-Abraham-Dirac vs Landau-Lifshitz radiation friction force in the ultrarelativistic electron interaction with electromagnetic wave (exact solutions)", Phys. Rev. E, 84, 056605 (2011).

Electron trajectories



Dependence of q_2 and q_3 (a,b), $q_\perp$ and $q_\parallel$ (c,d) and G_e and χ_e (e,f) on time for $\varepsilon_{\text{rad}} = 10^{-8}$ and $a_S = 4.1\times10^5$

(a,c) $a = 0.25\,\varepsilon_{rad}^{-1/3}$, (d,e) $a = 0.75\,\varepsilon_{rad}^{-1/3}$.

Integral scattering cross section

Stationary solutions

The energy flux reemitted by the electron is equal to $e(\mathbf{v}\cdot\mathbf{E})$,

which is $\approx \varepsilon_{\text{rad}} G_e(\chi_e) m_e c^2 \omega \gamma_e . \left(a q_\perp + a^2 q_\perp^2\right)$

The integral scattering cross section by definition equals the ratio

of the reemitted energy flux to the Poynting vector magnitude:

$$\sigma = \sigma_T G_e(\chi_e)\left(\frac{q_\perp}{a} + q_\perp^2\right)$$

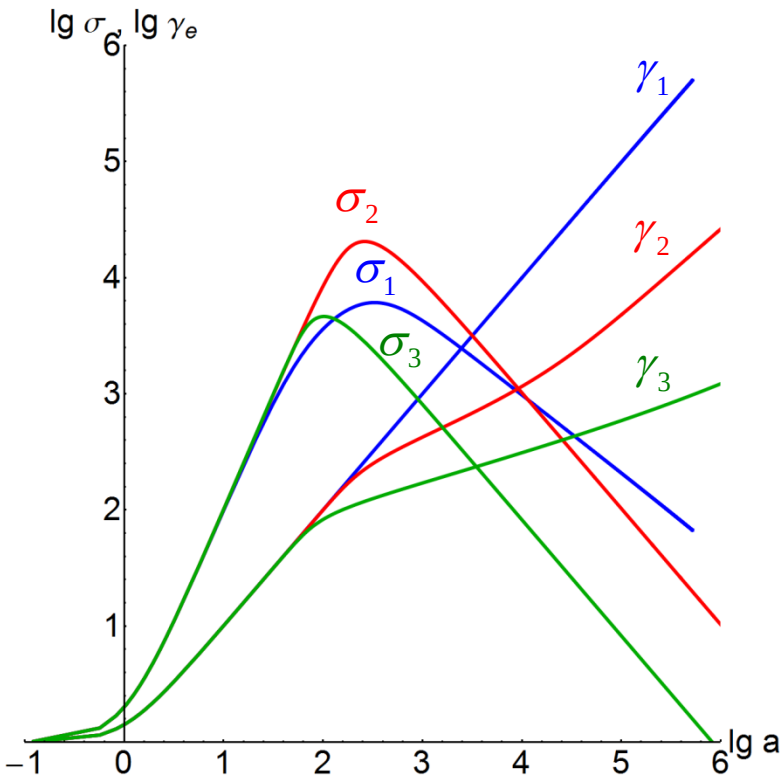
Here σ_T is the Thomson scattering cross section $\sigma_T = 8\pi r_e^2 / 3 = 6.65\times 10^{-25}\text{cm}^2$.

More convenient formula is

$$\sigma = \sigma_T\left(\frac{G_e \gamma_e^2}{1+\varepsilon_{rad}^2 G_e^2 \gamma_e^6}\right)$$

Ya. B. Zel'dovich, Sov. Phys. Usp. 18, 97 (1975)
S.V. Bulanov, T. Zh. Esirkepov, J. Koga, and T. Tajima, Plasma Phys. Rep. 30, 196 (2004)
T. Nakamura, J. K. Koga, T. Zh. Esirkepov, M. Kando, G. Korn, S. V. Bulanov, Phys. Rev. Lett. 108, 195001(2012)

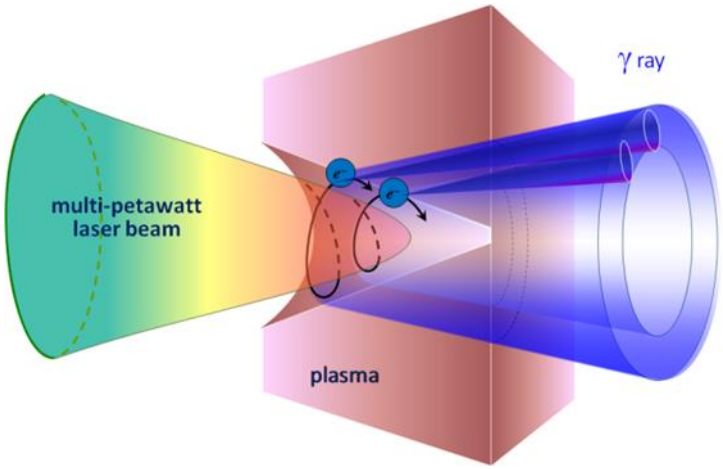
Cross section σ and electron energy γ_e v.s. EM wave amplitude



Dependences of $\lg(\sigma/\sigma_T)$ and of $\lg\gamma_e$ on $\lg a$
1) $\omega=\omega_1/12.5$, 2) $\omega=\omega_1$, 3) $\omega=12.5\,\omega_1$.

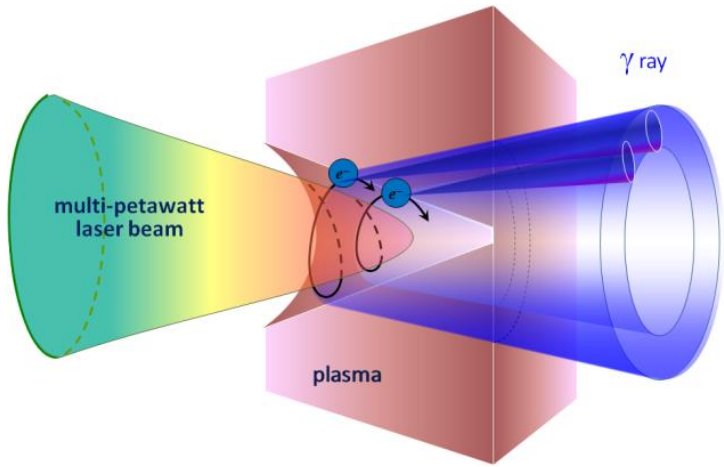
Nonlinear Thomson - Compton Scattering
Cross Section and Electron Gamma Factor
vs EM Field Amplitude.

High Power Gamma Flare Generation



High Power Gamma-Ray Source

Concept of high power gamma-flash generation:



Energy of photons emitted via NTS scales as $\mathcal{E}_\gamma = h\omega_\gamma \approx \hbar\omega\gamma_e^3$.

Electron energy is of the order of $m_e c^2 a$.

Photon energy is in the γ - ray range if $a > 10^2$ with intensity above $10^{22}\,\mathrm{W/cm^2}$.

Applications

- Photo-nuclear reactions
- Electron-positron pair creation
- Medicine
- Material sciences
- …….
- **Radiation safety**

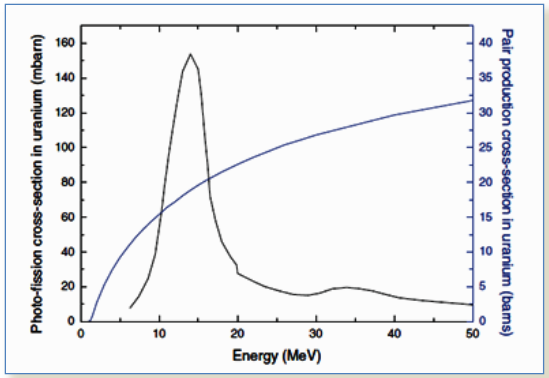


Photo-fission cross-section and pair production cross-section in uranium
[J. Galy, et al., New J. Phys. 9 (2007) 23]

T. Nakamura et al. Phys. Rev. Lett. 108, 195001 (2012)
K. V. Lezhnin, et al., Phys. Plasmas 25, 123105 (2018)

T. Nakamura, et al., PRL 108,195001 (2012)
C. P. Ridgers, et al., PRL 108, 165006 (2012)
……..
S. V. Bulanov, et al., PPR 41, 1 (2015)
Z. Gong, et al., Phys. Rev. E 95, 013210 (2017)
R. Capdepssus, et al., Sci. Rep. 8, 9155 (2018)
K. V. Lezhnin, et al., Phys. Plasmas 25, 123105 (2018)

Energy of emitted photon is

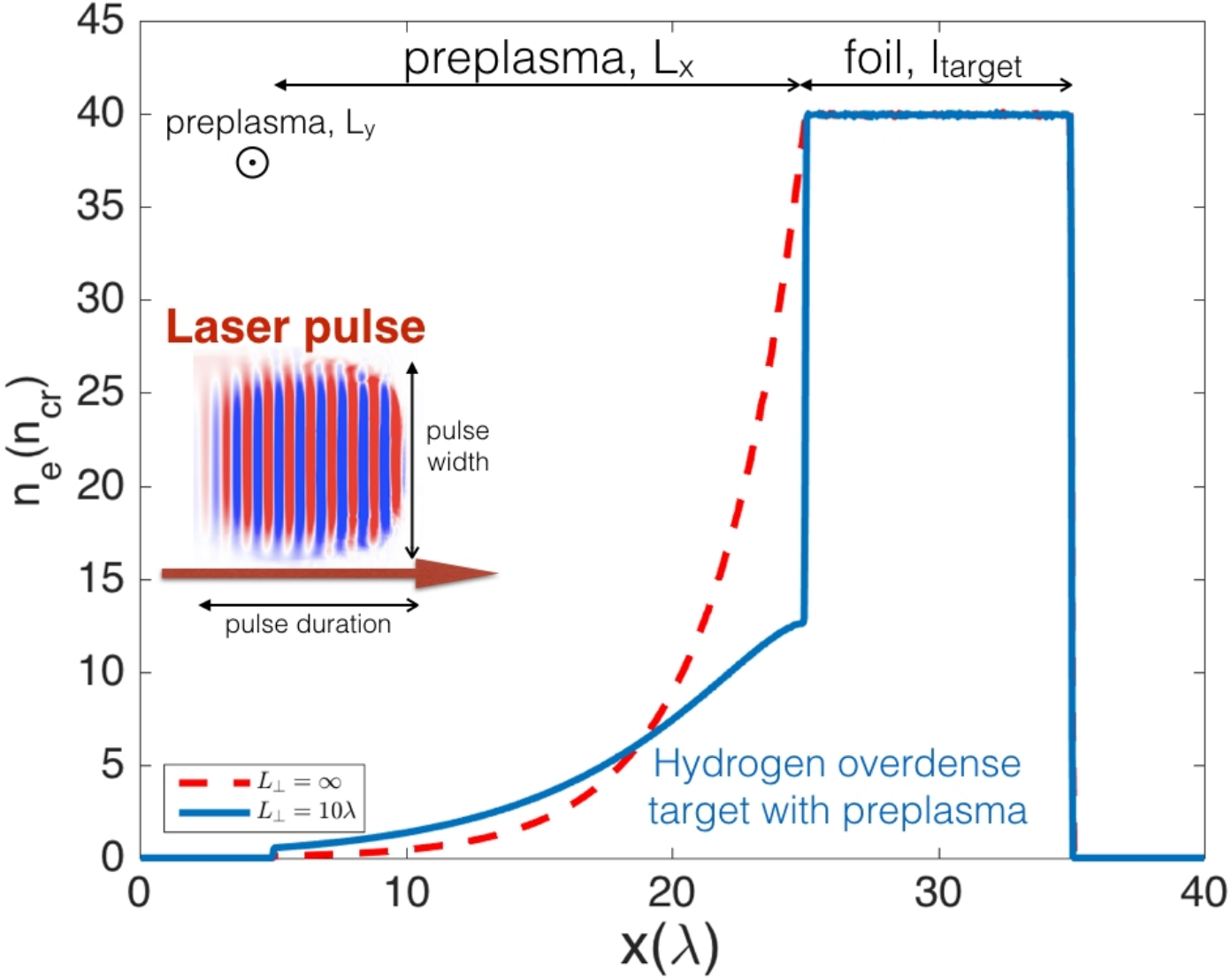
$$\hbar\bar{\omega}_\gamma = 0.3\hbar\bar{\omega}_0 a_0^3$$

Number of emitted photons/wave period

$$N_\gamma \approx \frac{3\pi}{4}\alpha a_0$$

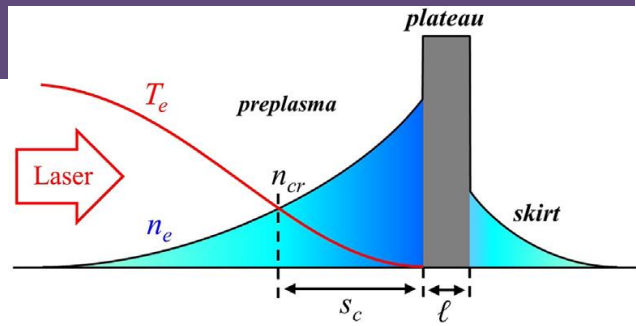
J. Koga, et al, Phys. Plasmas 12, 093106 (2005)
S.-Y. Chen, et al, Nature 396, 653 (1998)
G. Sarri, et al., Phys. Rev. Lett. 113, 224801 (2014)
W. Yan, et al, Nat. Photonics 11, 514520 (2017)

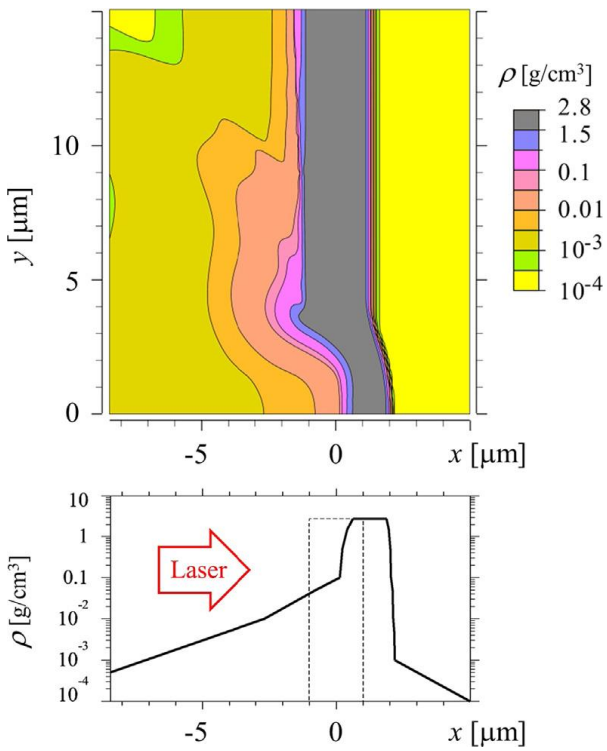
Simulation setup



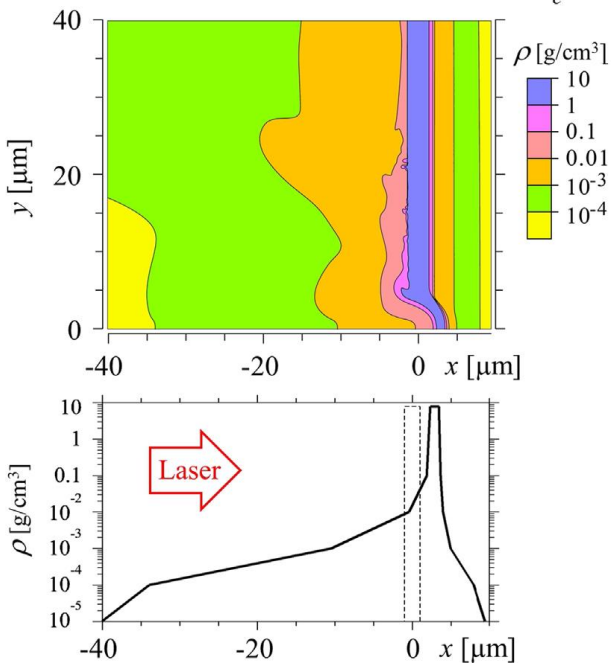
EPOCH: T. D. Arber et al., PPCF 57, 113001 (2015)

Preplasma





Density of aluminum target (g/cm^3) after 1ns of irradiation by prepulse with a 2 μm spot FWHM size, and intensity 2x10^{11} W/cm^2. Below is density along x-axis (solid curve) and initial target profile (dashed curve).

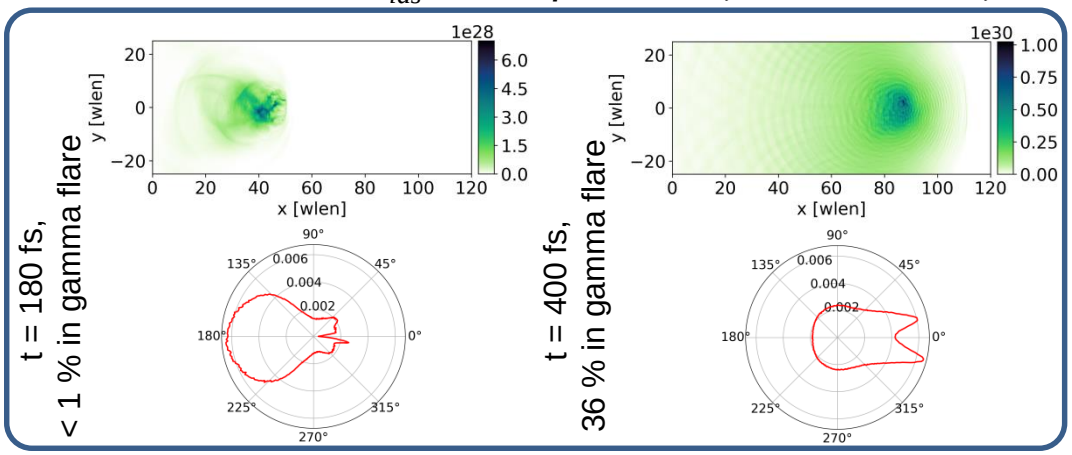


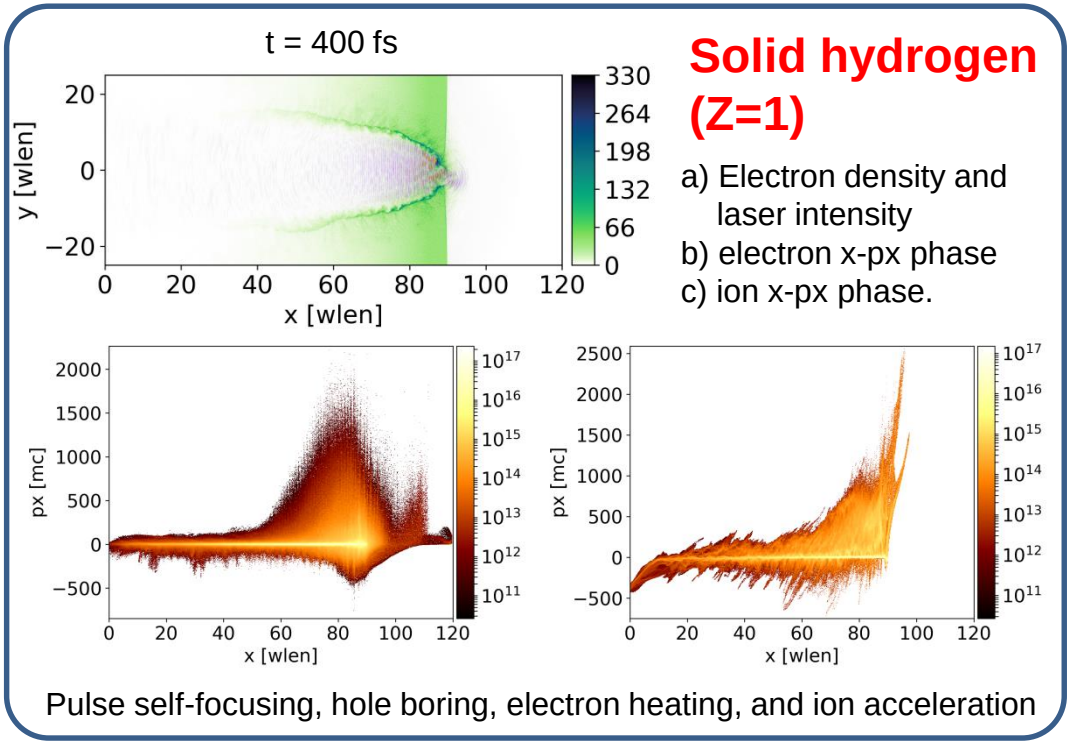
Density of iron target (g/cm^3) after 3ns of irradiation by prepulse with a 2 μm spot FWHM size, and intensity 2x10^{11} W/cm^2. Below is density along x-axis (solid curve) and initial target profile (dashed curve).

T. Zh. Esirkepov, et al." Prepulse and amplified spontaneous emission effects on the Interaction of a petawatt class laser with thin solid targets", Nuclear Instruments and Methods in Physics Research A 745, 150 (2014)

Optimal conditions for gamma flare generation (Hydrogen)

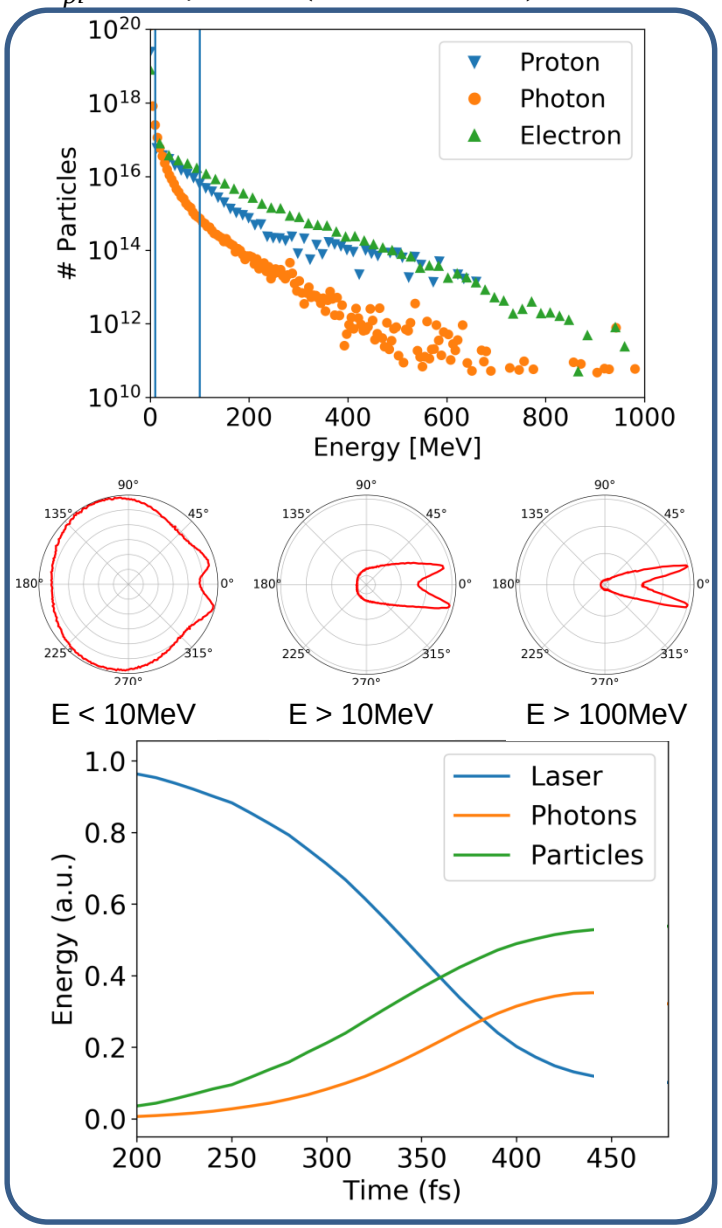
$10PW \quad \tau_{las} = 150\,fs \quad \lambda = 1\mu m \quad \varnothing = 2.5\mu m$





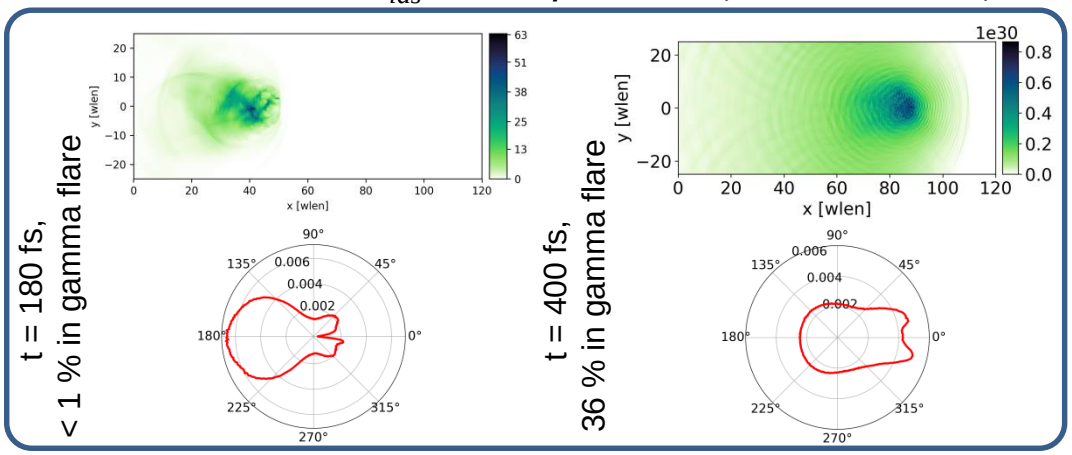
Pulse self-focusing, hole boring, electron heating, and ion acceleration

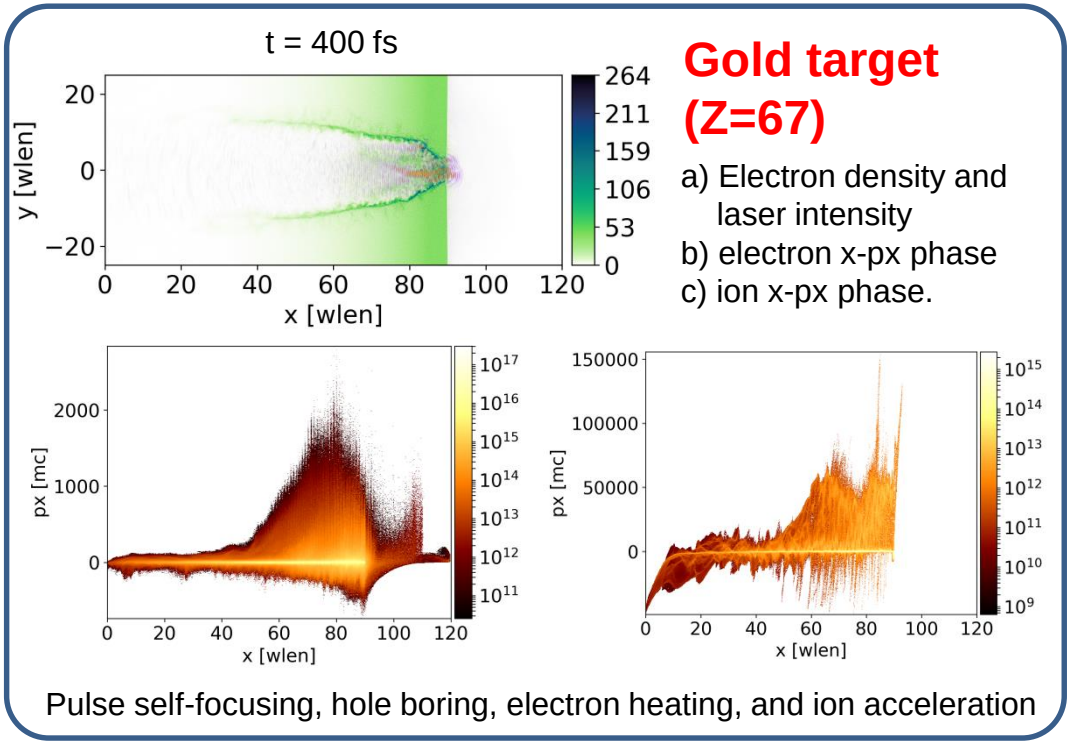
$L_{pl} = 80\mu m$ (ELI-BL L4)



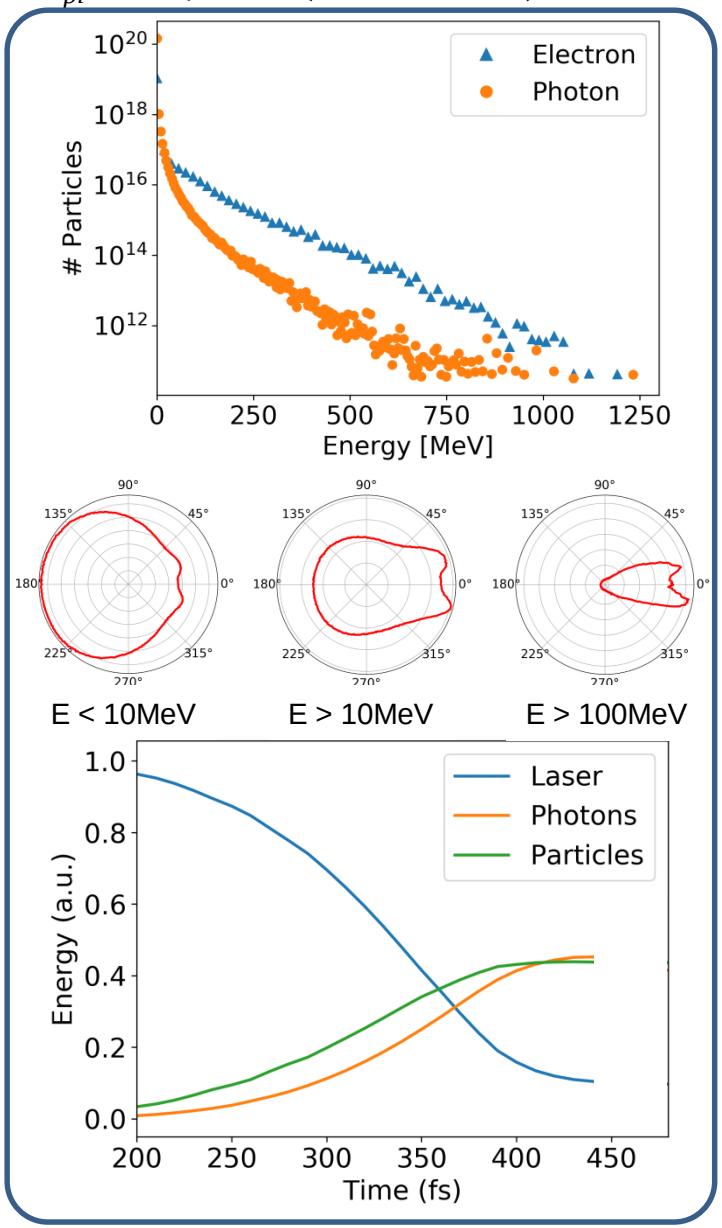
Optimal conditions for gamma flare generation (Gold target)

$10PW \quad \tau_{las} = 150\,fs \quad \lambda = 1\mu m \quad \varnothing = 2.5\mu m \qquad L_{pl} = 80\mu m$ (ELI-BL L4)



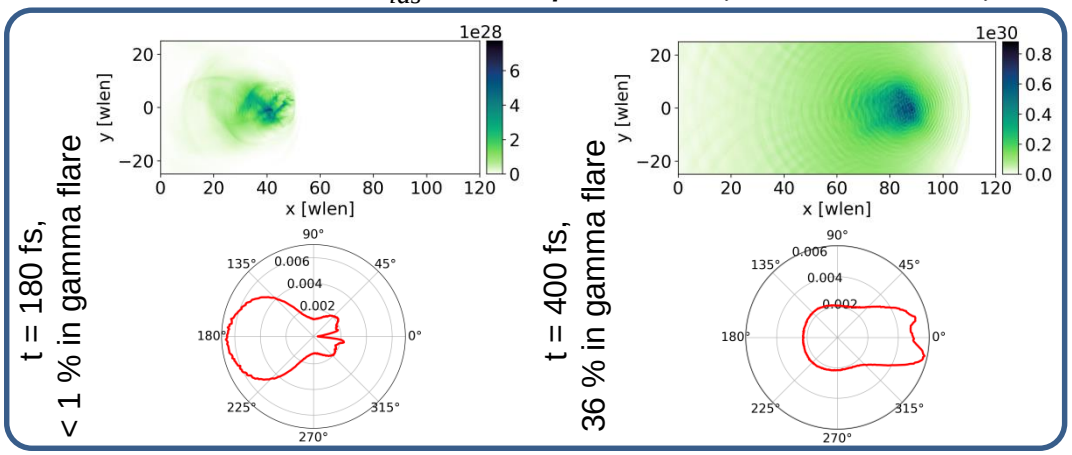


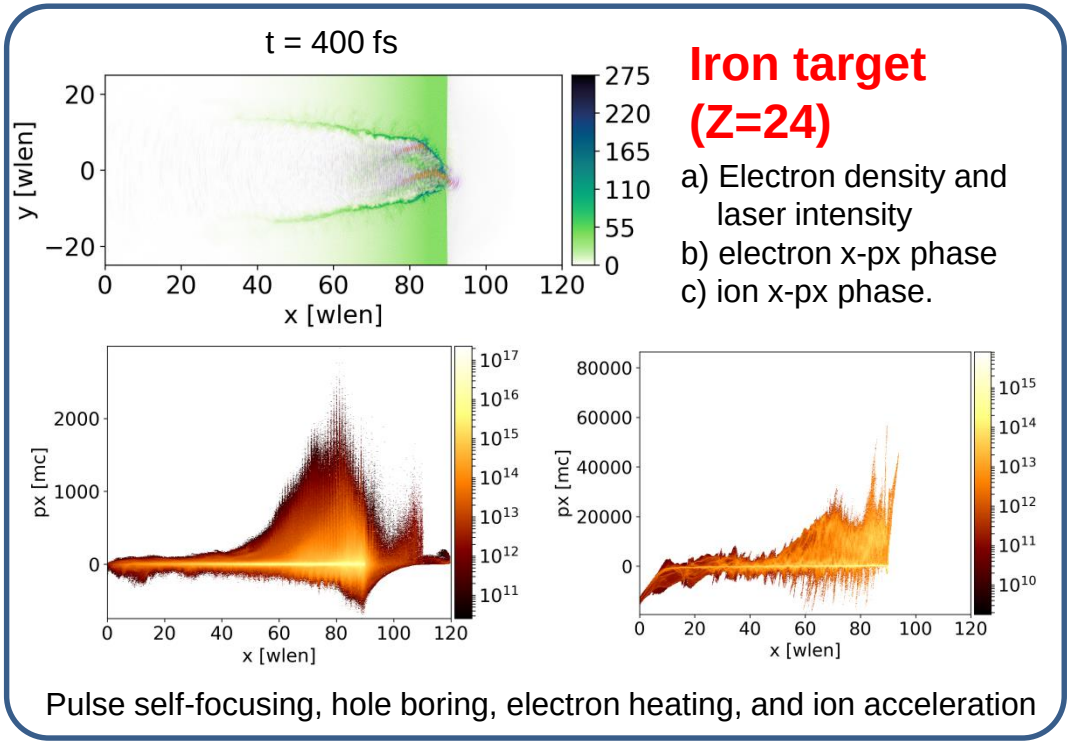
Pulse self-focusing, hole boring, electron heating, and ion acceleration



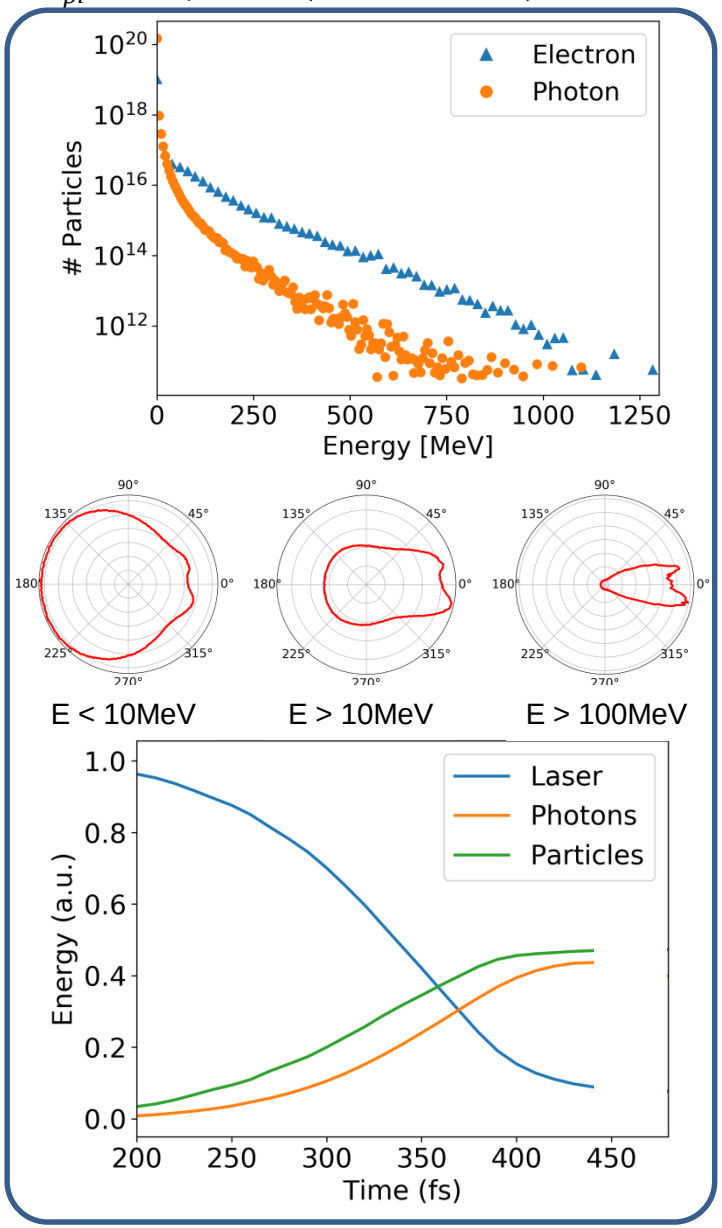
Optimal conditions for gamma flare generation (Iron target)

$10PW \quad \tau_{las} = 150\,fs \quad \lambda = 1\mu m \quad \varnothing = 2.5\mu m \qquad L_{pl} = 80\mu m$ (ELI-BL L4)



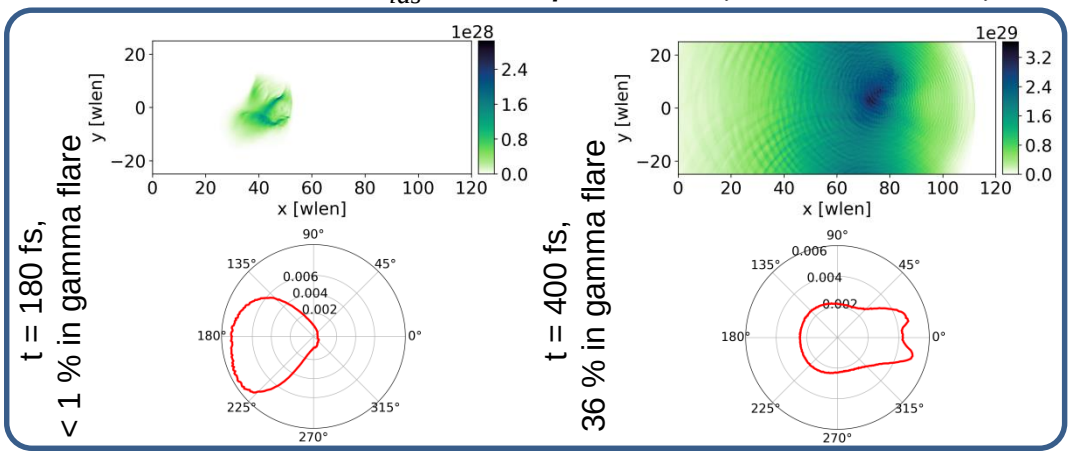


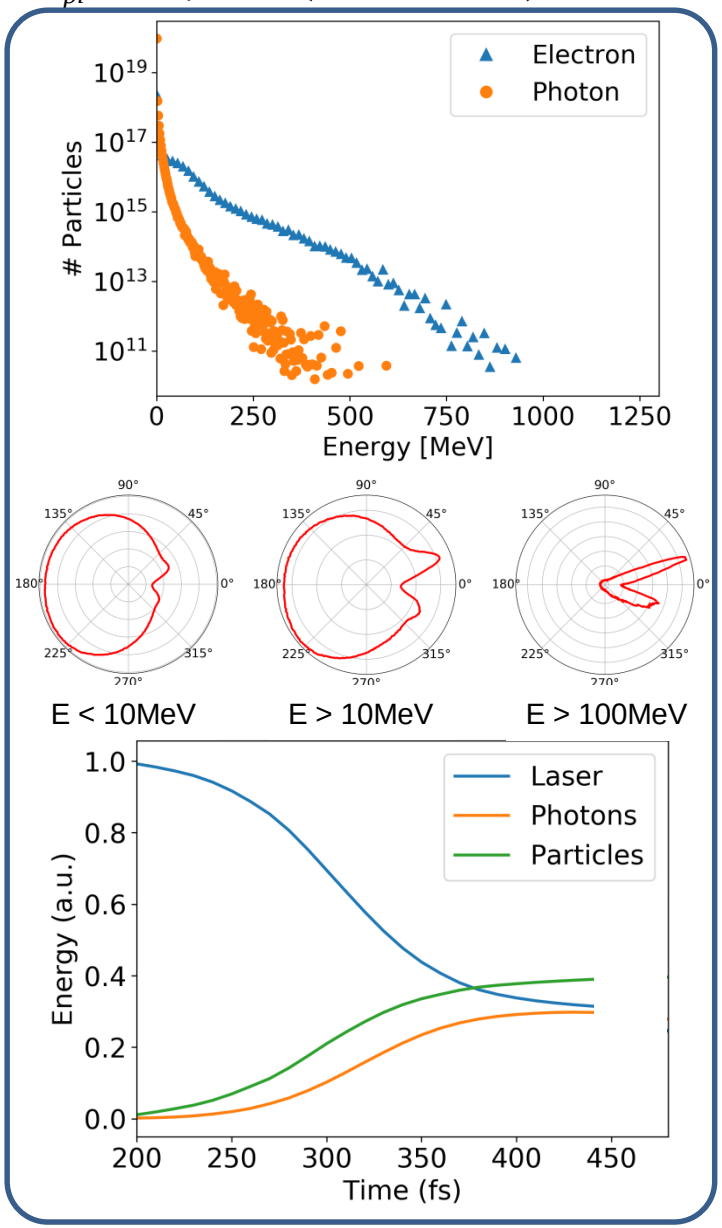
Pulse self-focusing, hole boring, electron heating, and ion acceleration

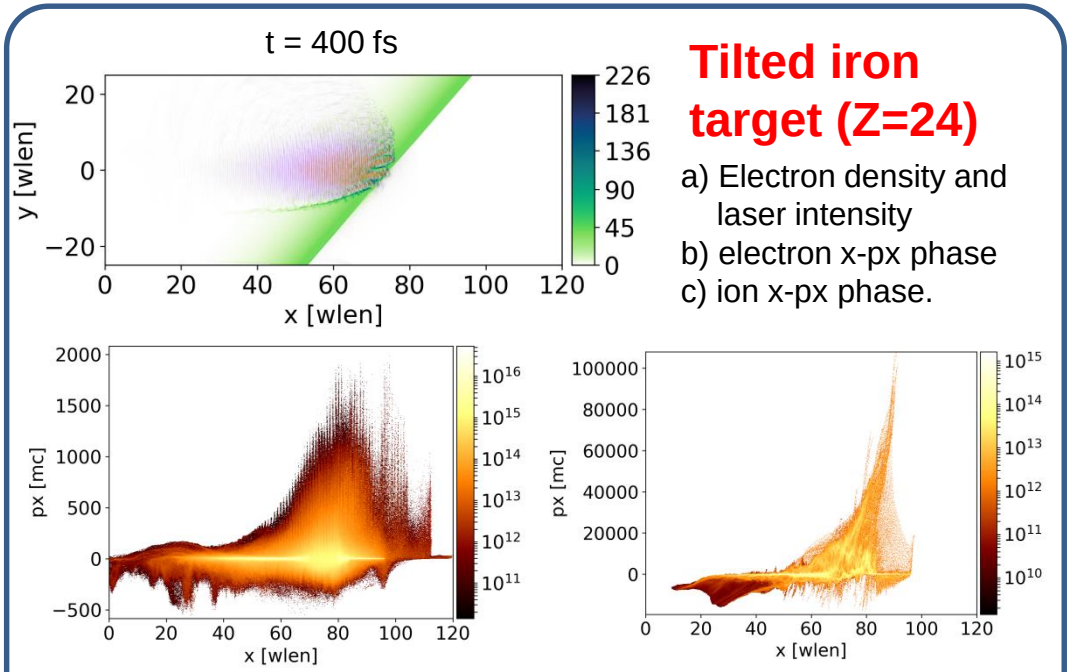


Optimal conditions for gamma flare generation (Tilted Iron target)

$10PW \quad \tau_{las} = 150\,fs \quad \lambda = 1\mu m \quad \varnothing = 2.5\mu m \qquad L_{pl} = 80\mu m$ (ELI-BL L4)







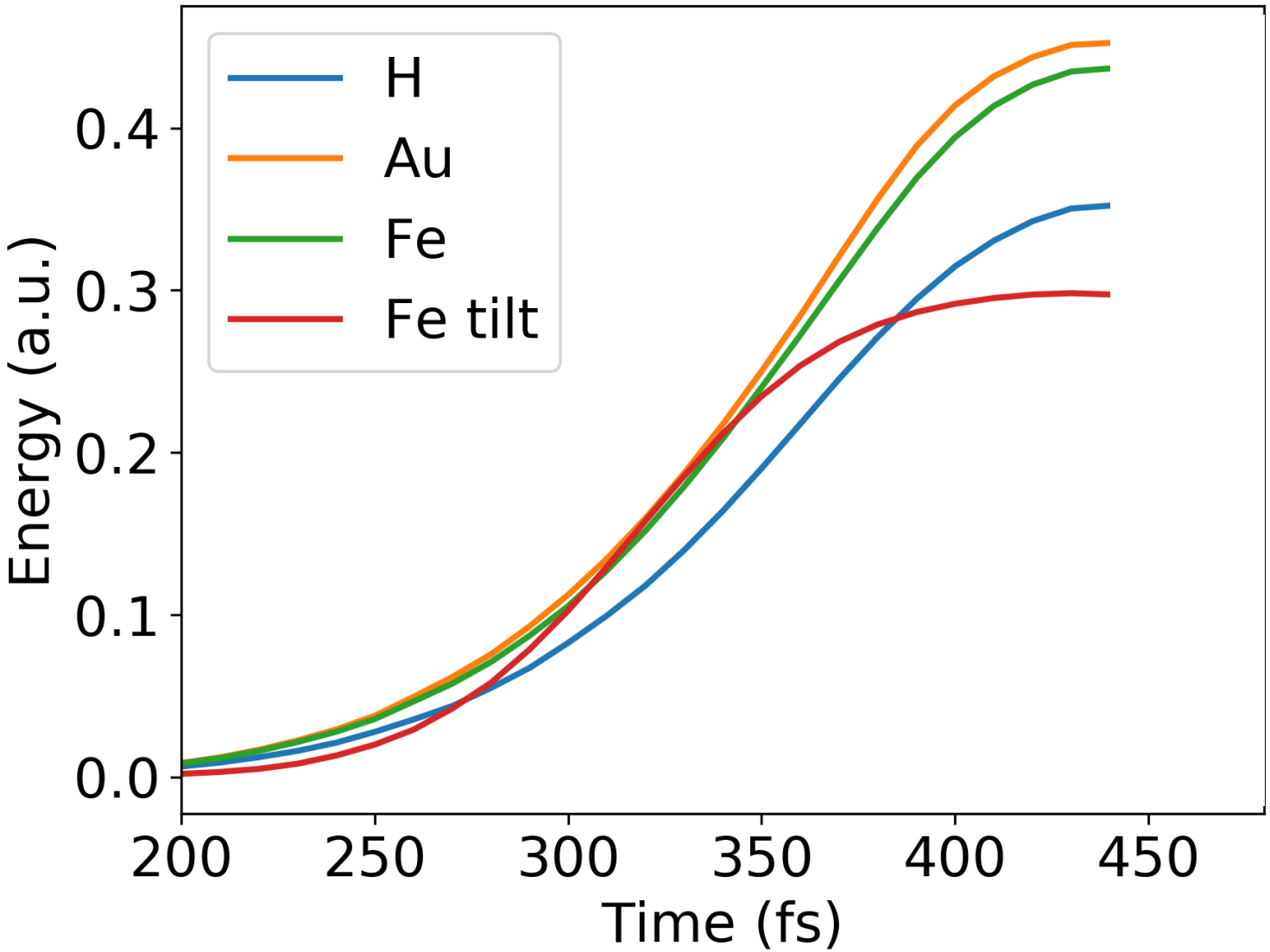
Tilted iron target (Z=24)

a) Electron density and laser intensity
b) electron x-px phase
c) ion x-px phase.

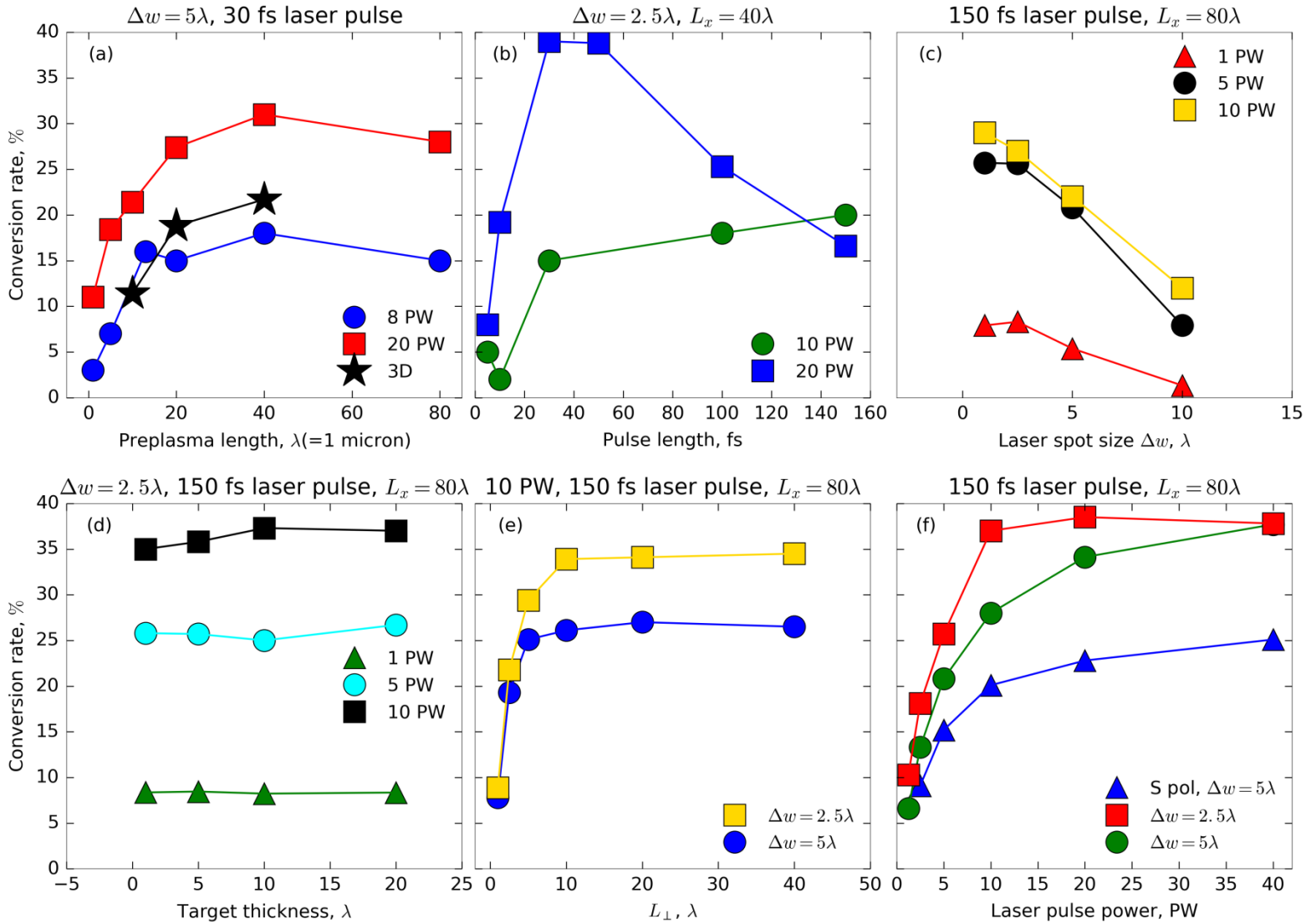
Pulse self-focusing, hole boring, electron heating, and ion acceleration

Photon generation efficiency

$10PW \quad \tau_{las} = 150\,fs \quad \lambda = 1\mu m \quad \varnothing = 2.5\mu m \qquad L_{pl} = 80\mu m$ (ELI-BL L4)

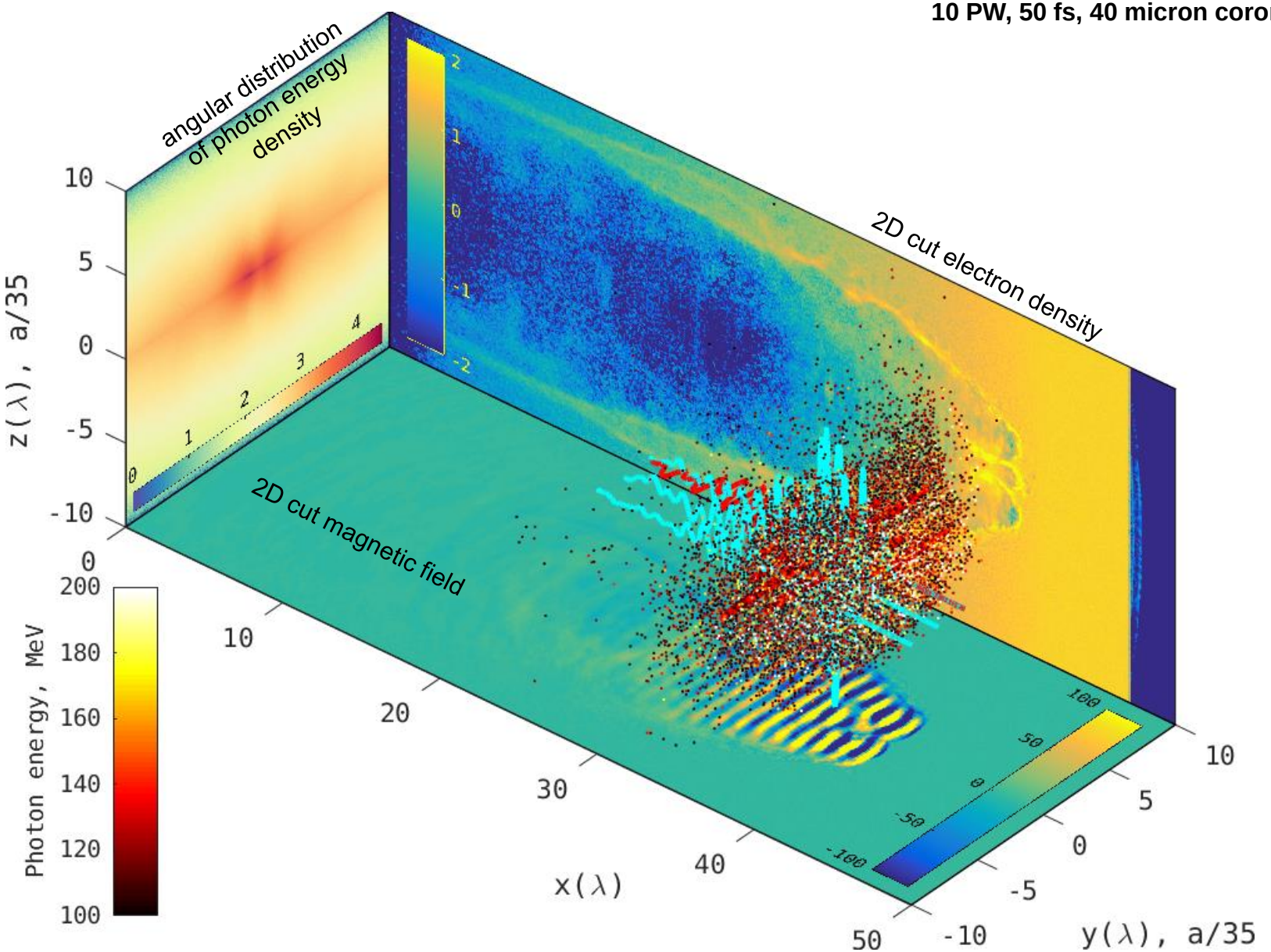


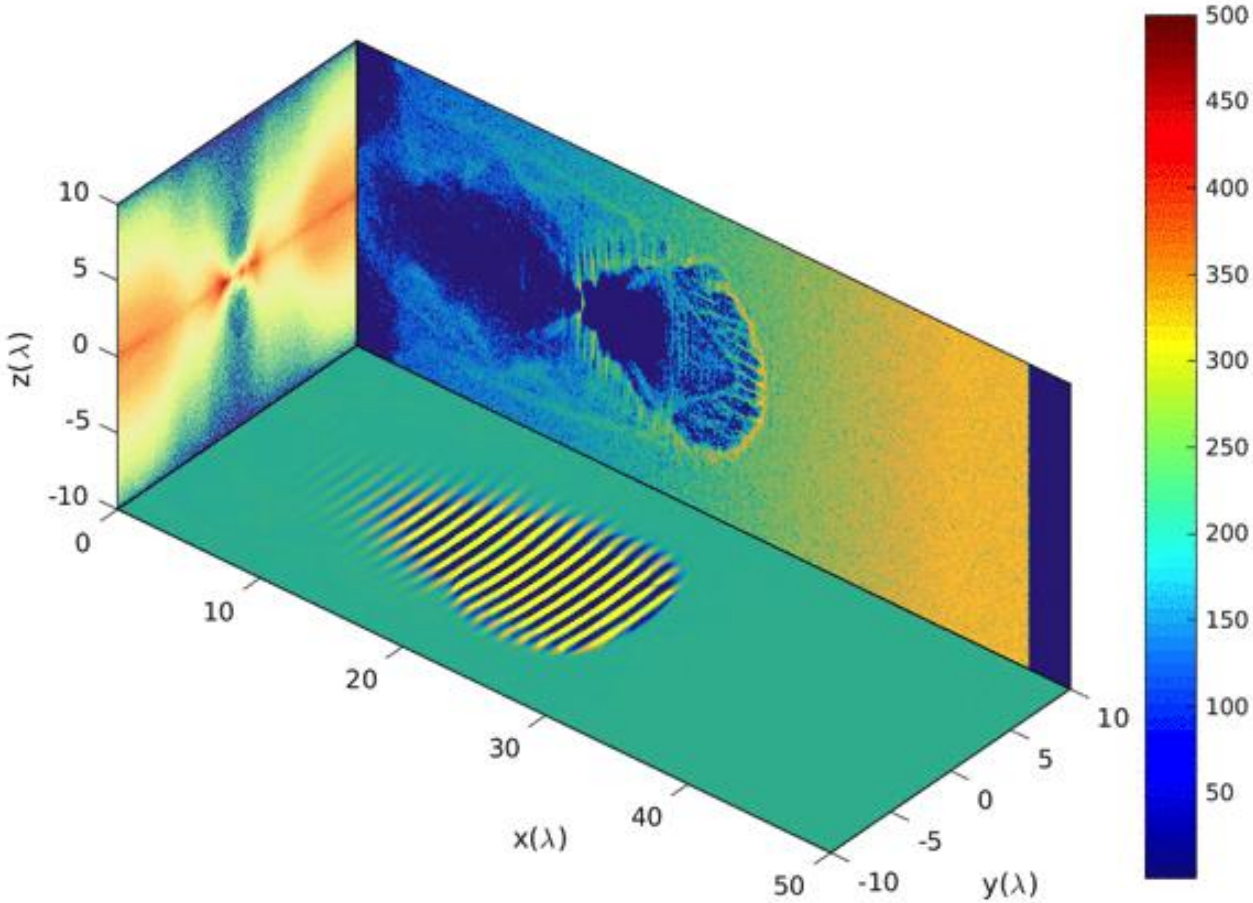
Multi-parametric analysis of target & laser pulse



3D simulations of gamma flare generation

10 PW, 50 fs, 40 micron corona




z(λ)
10
5
0
-5
-10
0
10
20
30
40
50
x(λ)
-10
-5
0
5
10
y(λ)
500
450
400
350
300
250
200
150
100
50

Kinematics of inverse multi-photon Compton scattering in vacuum

Nonlinear Thomson scattering $\hbar\omega_\gamma \approx 0.3\hbar\omega_0 a_0^3$ i.e. $N_{ph} = a_0^3$

Recoil at $\hbar\omega_\gamma \approx mc^2 a_0$ imposes constraint on the photon energy :

$$a_0 < a_m = \sqrt{m_e c^2 / \hbar\omega_0}$$

The energy - momentum conservation equates the sum of electron and photon energy - momentum before and after scattering

$$m_e c^2\gamma + \hbar\omega_\gamma = m_e c^2\gamma_0 + N_{ph}\hbar\omega_0, \quad \boldsymbol{p} + \hbar\boldsymbol{k}_\gamma = \boldsymbol{p}_0 + N_{ph}\hbar\boldsymbol{k}_0$$

In vacuum $\omega^2 = \boldsymbol{k}^2 c^2$. It yields

$$\hbar\omega_\gamma = \frac{N_{ph}\hbar\omega_0(p_{\|,0}c + m_e c^2\gamma_0)}{N_{ph}\hbar\omega_0 + m_e c^2\gamma_0 - p_{\perp,0}c\sin\theta + (\hbar\omega_0 - p_{\|,0}c)\cos\theta}$$

Maximum photon energy $\hbar\omega_\gamma \approx m_e c^2 p_{\|,0}$ energy in the head - on collision.

In the receding configuration, for co - propagating electron and laser pulse the scattered photon energy is well bellow the incident photon energy.

F. Ehlotzky, K. Krajewska, and J. Z. Kaminski, Rep. Prog. Phys. 72, 046401 (2009)
C. Bula, et al., Phys. Rev. Lett. 76, 3116 (1996)

Inverse multi-photon Compton scattering in collisionless plasma

The energy - momentum conservation equates the sum of electron and photon energy - momentum before and after scattering

$$m_e c^2 \gamma + \hbar\omega_\gamma = m_e c^2 \gamma_0 + N_{ph}\hbar\omega_0, \quad \boldsymbol{p} + \hbar \boldsymbol{k}_\gamma = \boldsymbol{p}_0 + N_{ph}\hbar \boldsymbol{k}_0$$

In collisionless plasma $\omega^2 = \boldsymbol{k}^2 c^2 + \bar{\omega}_{pe}^2$ where $\bar{\omega}_{pe}^2 = 4\pi n e^2 / m_e \sqrt{1 + a_0^2}$

Assuming $\omega_0 = \bar{\omega}_{pe}$ (electron - e.m. wave interaction at critical surface) we obtain

$$\hbar\omega_\gamma = \frac{N_{ph}\hbar\omega_0(\hbar\omega_0 + m_e c^2 \gamma_0)}{N_{ph}\hbar\omega_0 + m_e c^2 \gamma_0 - p_{\perp,0} c \sin\theta - p_{\|,0} c \cos\theta}$$

Maximum photon energy $\hbar\omega_\gamma \approx m_e c^2 \gamma_0$.

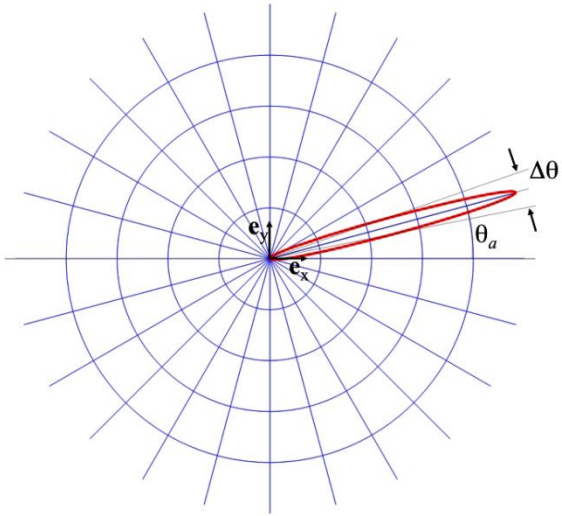
Electron momentum : $p_{\perp,0} = m_e c a_0, \quad p_{\|,0} = m_e c a_0^2 / 2$

Gamma photons are emitted at the angle θ_a :

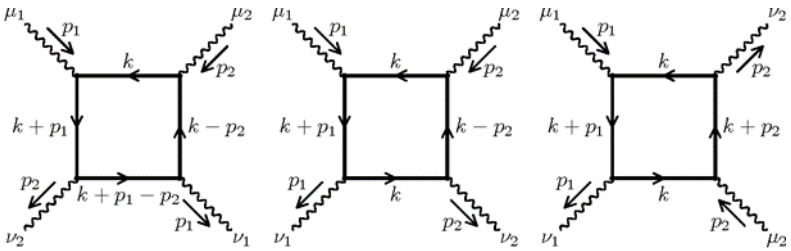
$$-a_0 / p_{\|,0} < \theta_a < a_0 / p_{\|,0}$$

being confined within the angle

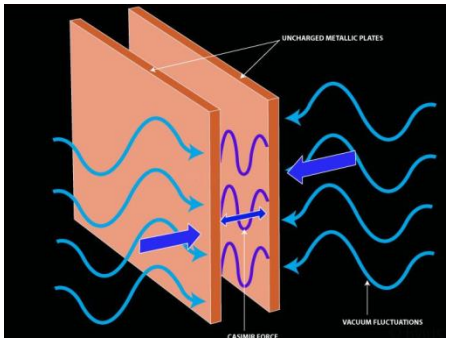
$$\Delta\theta_a \approx \sqrt{(1 + a_0^2) m_e^2 c^2 / p_{\|,0}^2 + N_{ph}\hbar\omega_0 / m_e c}$$



Nonlinear Electromagnetic Waves in the QED Vacuum

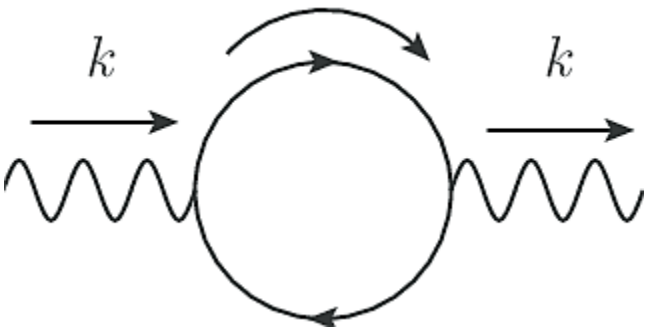


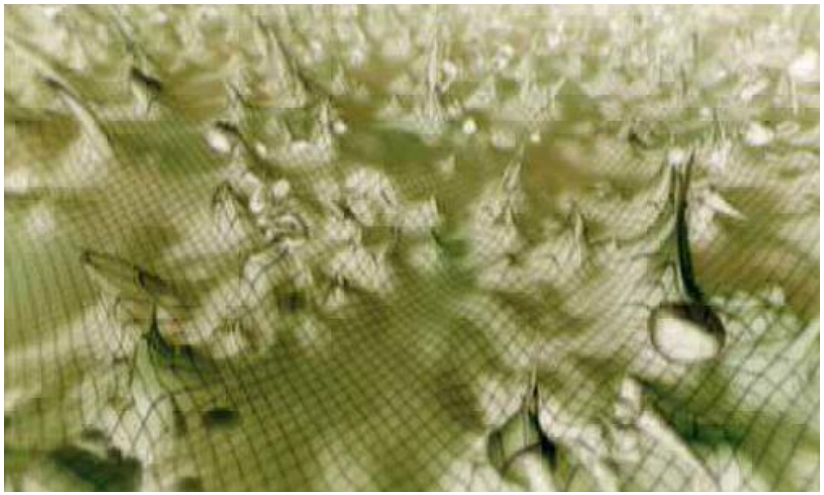
Vacuum Polarization



QFT: Quantum Electrodynamics

- **Vacuum is not a void medium**
- **Creation and annihilation of electron-positron pairs**
- **Casimir effect, Delbruck scattering, photon-photon-scattering, …**
- **Fluctuations**

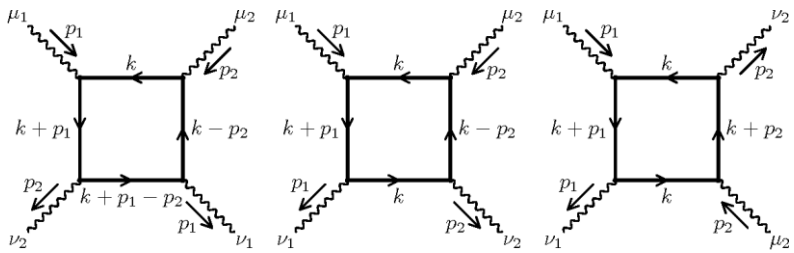




B. Green, The Elegant Universe (2003)

Photon–Photon Scattering

In the QED, photon-photon scattering occurs via creation-annihilation of virtual electron-positron pairs by two initial photons followed by annihilation of the pairs into final photons

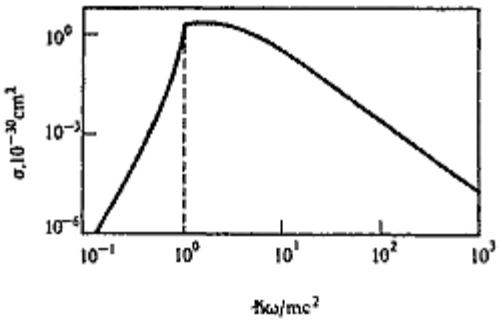


R. Karplus and M. Neuman, Phys. Rev. 83, 776 (1951)

ATLAS Collaboration, Nature Physics 13, 852 (2017)

Dependence of the photon-photon scattering cross section on the photon frequency (assume that $\omega_1 = \omega_2 = \omega_\gamma$)

$$\sigma_{\gamma\gamma} = \begin{cases} \left(\dfrac{973}{10125\pi}\right)\alpha^2 r_e^2 \left(\dfrac{\hbar\omega_\gamma}{m_e c^2}\right)^6 & \text{for } \hbar\omega_\gamma << m_e c^2 \\ \left(\dfrac{3}{12\pi}\right)^2 \alpha^2 r_e^2 \left(\dfrac{m_e c^2}{\hbar\omega_\gamma}\right)^2 & \text{for } \hbar\omega_\gamma >> m_e c^2 \end{cases}$$



Pair Creation via the Breit-Wheeler Process

Electron-positron pair creation via the Breit-Wheeler process has the cross section

$$\sigma_{\gamma\gamma\to ep} = \frac{1}{2}\pi r_e^2 (1-\beta_e^2)\left\{(3-\beta_e^4)\ln\left(\frac{1+\beta_e}{1-\beta_e}\right) - 2\beta_e(2-\beta_e^2)\right\}$$

where

$$\beta_e = \sqrt{1 - 2m_e^2 c^4 / \hbar^2 \omega_1 \omega_2 (1-\cos\varphi)}$$

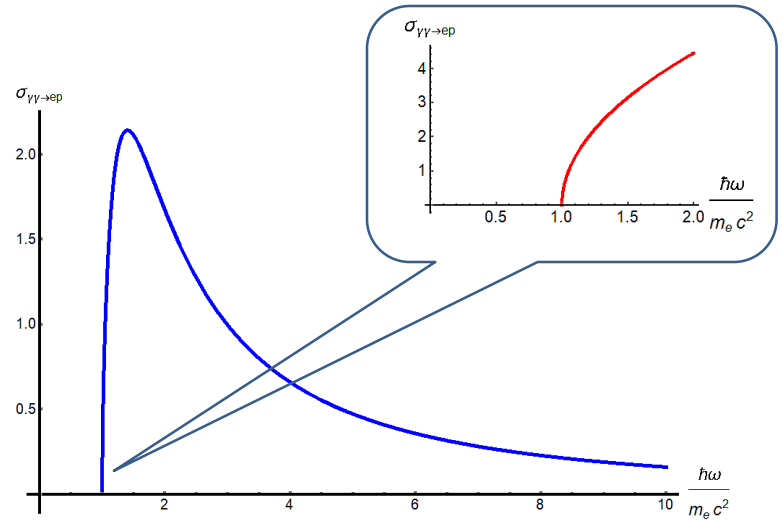
is Lorentz invariant

Near the threshold when $\beta_e \le 1$

$$\omega_1\omega_2(1-\cos\varphi) \ge 2m_e^2 c^4 / \hbar^2$$

the cross section is given by

$$\sigma_{\gamma\gamma\to ep} = \pi r_e^2 \sqrt{\frac{1}{\beta_e} - 1}$$



G. Breit and J. A. Wheeler, Physical Review 46, 1087 (1934)
X Ribeyre, E d'Humi`eres, S Jequier and V T Tikhonchuk, PPCF 60, 104001 (2018)

Photon-Photon Scattering: the Number of Scattered Photons

The number of photons in the electromagnetic pulse with amplitude E in the 4-volume λ^3/ω

$$N_\gamma = \frac{E^2\lambda^3}{4\pi\hbar\omega}$$

In optimal regime (**maximal luminosity**) the number of scattering events per 4-volume is proportional to the scattering cross section, to the product of the photon numbers in colliding photon bunches, and it is inverse proportional to the square of the wavelength

$$N_{\gamma\gamma} = \sigma_{\gamma\gamma}\frac{N_\gamma N_\gamma}{\pi\lambda^2}$$

This yields

$$N_{\gamma\gamma} = \left(\frac{973}{10125\pi}\right)\alpha^2\left(\frac{E}{E_S}\right)^4 \propto \left(\alpha\frac{I}{I_S}\right)^2$$

Here E_S is the Schwinger (Sauter, Bohr, …) field equal to

$$E_S = \frac{m_e^2c^3}{e\hbar}$$

where $\alpha = e^2/\hbar c \approx 1/137$: e.m. field intensity $I_S = cE_S^2/4\pi = 10^{29}W/cm^2$

QED VACUUM IS DISPERSIONLESS MEDIUM

Heisenberg-Euler Lagrangian

In the long wavelength and low frequency approximation ($|\partial_\mu A_\nu| / |A_\mu| << \lambda\!\!\!^{-1}_C$)
the Lagrangian describing the electromagnetic field in vacuum is

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}'$$

where

$$\mathcal{L}_0 = -\frac{1}{16\pi} F^{\mu\nu} F_{\mu\nu}$$

with

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

gives the Maxwell equations.

The Heisenberg-Euler term

$$\mathcal{L}' = -\frac{m_e^4}{8\pi^2} \int_0^\infty \frac{\exp(-\eta)}{\eta^3} \left\{ 1 - \frac{\eta^2}{3}\left(\mathfrak{a}^2 - \mathfrak{b}^2\right) - \left[\eta\mathfrak{a}\cot\left(\eta\mathfrak{a}\right)\right]\left[\eta\mathfrak{b}\coth\left(\eta\mathfrak{b}\right)\right] \right\} d\eta$$

Here the invariants *a* and *b* can be expressed in terms the Poincare invariants

$$\mathfrak{F} = F^{\mu\nu} F_{\mu\nu} \text{ and } \mathfrak{G} = F^{\mu\nu} \tilde{F}_{\mu\nu}$$

(dual tensor equals $\tilde{F}^{\mu\nu} = \varepsilon^{\mu\nu\rho\sigma} F_{\rho\sigma}$) as

$$\mathfrak{a} = \sqrt{\sqrt{\mathfrak{F}^2 + \mathfrak{G}^2} + \mathfrak{F}} \text{ and } \mathfrak{b} = \sqrt{\sqrt{\mathfrak{F}^2 + \mathfrak{G}^2} - \mathfrak{F}}$$

Here we use the units $c = \hbar = 1$, and the e.m. field is normalized on the QED critical field E_S.

W. Heisenberg and H. Euler, Zeit. fuer Phys. 98, 714 (1936)

Weak Field Approximation

The Heisenberg-Euler term in the weak field approximation we have

$$\mathcal{L}' = \frac{\kappa}{4}\left\{\left(F^{\mu\nu}F_{\mu\nu}\right)^2 + \frac{7}{4}\left(F^{\mu\nu}\tilde{F}_{\mu\nu}\right)^2 + \frac{90}{315}\left(F^{\mu\nu}F_{\mu\nu}\right)\left[\left(F^{\mu\nu}F_{\mu\nu}\right)^2 + \frac{13}{16}\left(F^{\mu\nu}\tilde{F}_{\mu\nu}\right)^2\right]\right\}$$

With the constant

$$\frac{\kappa}{4} = \left(\frac{e^4}{360\pi^2}\right)m_e^4$$

The first two terms describe four interacting photons and the last two terms correspond to six photon interaction.

$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}'$ gives equations of the nonlinear electrodynamics in the dispersionless medium with refraction index depends on the electromagnetic field.

Dirac's Light Cone Coordinates

Below we use the light cone coordinates

$$x^+ = (x+t)/\sqrt{2}, \quad x^- = (x-t)/\sqrt{2}$$

Here $c = 1$.

The variables (x,t) in the lab. frame of reference are related to (x',t') in the boosted frame moving with the velocity β as

$$x' = x\cosh\eta - t\sinh\eta, \qquad t' = t\cosh\eta - x\sinh\eta$$

where $\eta = \sqrt{(1+\beta)/(1-\beta)}$

The Lorentz transform of the light-cone variables is

$$x'^+ = (x'+t')/\sqrt{2} = e^{-\eta}(x+t)/\sqrt{2} = e^{-\eta}x^+, \quad x'^- = (x'-t')/\sqrt{2} = e^{+\eta}(x-t)/\sqrt{2} = e^{+\eta}x^-$$

For derivatives we have $(\partial_\pm)' = e^{\pm\eta}\partial_\pm$

Introducing the fields $u = \partial_- a$ and $w = \partial_+ a$

related to the electric $e = -\partial_t a$ and magnetic $b = \partial_x a$ field by

$$u = (e-b)/\sqrt{2} \quad \text{and} \quad w = (e+b)/\sqrt{2}$$

we obtain

$$u' = e^{-\eta}u \quad \text{and} \quad w' = e^{+\eta}w$$

The field product $uw = (e^2 - b^2)/2$

$$u'w' = uw$$

is Lorentz invariant.

Finite Amplitude Counter-Propagating EM Waves

The e.m. waves have the same polarization ($\boldsymbol{A} = A\boldsymbol{e}_z$)

We use the light cone coordinates

$$x^+ = (x+t)/\sqrt{2}, \quad x^- = (x-t)/\sqrt{2}$$

Propagation along the x axis,

$$A = Wx^+ + a(x^+, x^-)$$

where Wx^+ is the crossed e.m. field

and $a(x^+, x^-)$ is the 4 potential for counter-propagatying e.m. wave.

H-E Lagrangian can be cast to

$$\mathcal{L} = -\frac{1}{4}\left[(W+w)u - \varepsilon_2 (W+w)^2 u^2 - \varepsilon_3 (W+w)^3 u^3\right]$$

with the field components $u = \partial_+ a, \quad w = \partial_- a$

Lagrangian variation $\partial_+ (\partial L / \partial u) + \partial_- (\partial L / \partial w) = 0$

yields system of nonlinear equations for electromagnetic wave

$$\partial_+ w - \partial_- u = 0$$

$$\left[1 - 4\varepsilon_2 (W+w)u - 9\varepsilon_3 (W+w)^2 u^2\right]\partial_+ u - \left[\varepsilon_2 (W+w)^2 + 3\varepsilon_3 (W+w)^3 u\right]\partial_- u$$
$$-\left[\varepsilon_2 u^2 + 3\varepsilon_3 (W+w) u^3\right]\partial_+ w = 0$$

with $\varepsilon_2 = (2/45\pi)\alpha$ and $\varepsilon_3 = (32/315\pi)\alpha$

Phase and Group Velocity

The e.m. wave propagates along the x direction with velocity

$$v_W = \frac{1-\varepsilon_2 W^2}{1+\varepsilon_2 W^2} \approx 1 - 2\varepsilon_2 W^2$$

The phase velocity of the wave is equal to the group velocity,
i.e. the vacuum is dispersionless medium.
The normalized $\beta_{ph} = \beta_g = v_W / c$ are less than unity, i.e.,
the wave phase (group) velocity is below the speed of light in vacuum.

Corresponding dispersion equation for the e.m. wave frequency and vave number has a form

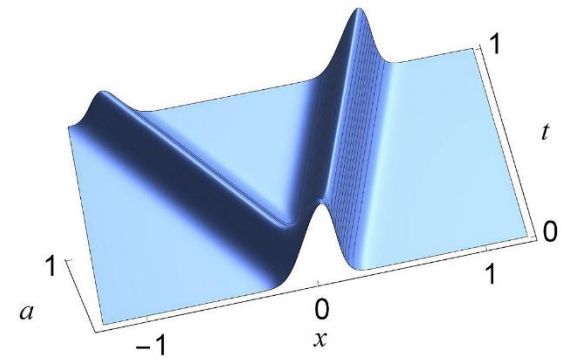
$$(\omega + kc)(\omega - kc + 2kc\varepsilon_2 W^2) = 0$$

Refraction index of the QED vacuum $n = 1 + 2\varepsilon_2 W^2$
is greater than unity with $\Delta n = 2\varepsilon_2 W^2$

Bialynicka - Birula & Bialynicki - Birula (1970)

Dittrich & Gies (2000)

Marklund & Shukla (2006)



Electromagnetic waves
in the (x,t) plane

Electromagnetic Riemann Wave in Vacuum

Assuming the solution of nonlinear wave equations in the Riemann wave form: $w = w(u)$, we obtain

$$\left[\varepsilon_2 u^2 + 3\varepsilon_3 (W+w) u^3\right] J^2 - \left[1 - 4\varepsilon_2 (W+w) u - 9\varepsilon_3 (W+w)^2 u^2\right] J + \left[\varepsilon_2 (W+w)^2 + 3\varepsilon_3 (W+w)^3 u\right] = 0$$

Here $J = \dfrac{dw}{du}$ is the Jacobian.

For small but finite u it can be found to be equal to

$$J = \varepsilon_2 W^2 + \left(4\varepsilon_2^2 + 3\varepsilon_3\right) W^3 u + \ldots$$

We arrive to the equation for nonlinear e.m. wave in the form

$$\partial_+ u + \left(\varepsilon_2 W^2 + \left(4\varepsilon_2^2 + 3\varepsilon_3\right) W^3 u\right) \partial\_u = 0$$

In the x, t variables it can be rewritten as

$$\partial_t u + \left(v_W - 2\left(4\varepsilon_2^2 + 3\varepsilon_3\right) W^3 u\right) \partial_x u = 0$$

Wave Breaking (Rarefaction Wavebreak)

This is the Hopf equation. We rewrite it as

$$\partial_t \bar{u} + \left(v_W - \bar{u}\right)\partial_x \bar{u} = 0 \qquad (*)$$

with

$$\bar{u} = 2\left(4\varepsilon_2^2 + 3\varepsilon_3\right)W^3 u$$

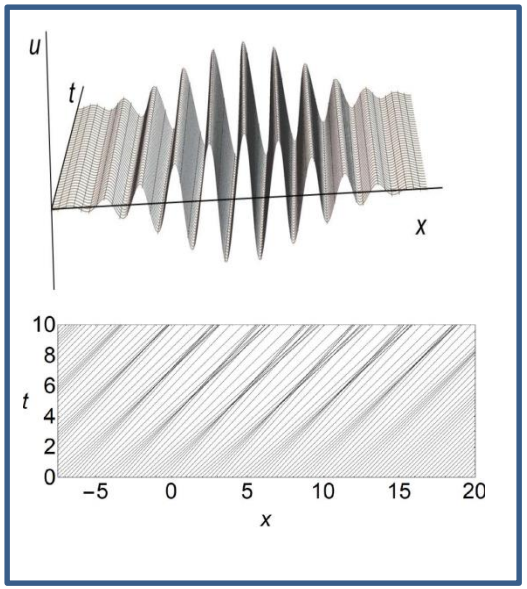
For given initial condition $\bar{u}_0\left(x\right)$ its solution in implicit form is given by

$$\bar{u} = \bar{u}_0\left(x - \left(v_W - \bar{u}\right)t\right) \qquad (**)$$

It can be obtained by integrating along the characteristics:

$$\frac{dx}{dt} = v_W - \bar{u}, \quad \frac{d\bar{u}}{dt} = 0$$

with the solution $\bar{u} = \bar{u}_0\left(x_0\right),\ x = x_0 + \left(v_W - \bar{u}_0(x_0)\right)t$



Calculating the field gradient we find that

$$\partial_x u = \frac{\partial_{x_0} u_0(x_0)}{1 - 2\left(4\varepsilon_2^2 + 3\varepsilon_3\right)W^3 \partial_{x_0} u_0(x_0) t}$$

At $t = t_{br}$

$$t_{br} = \frac{1}{2\left(4\varepsilon_2^2 + 3\varepsilon_3\right)W^3 \partial_{x_0} u_0(x_0)}$$

the denominator vanishes - the wave breaks (**gradient catastrophe**)

Lutzky , Toll, Phys. Rev. 113, 1649 (1959); Böhl, King, Ruhl, Phys. Rev. A 92, 032115 (2015);
H. Kadlecova, G. Korn, SVB, Phys. Rev. D 99, 036002 (2019)

EM Shock Wave in QED Vacuum

Expression (**) describes high order harmonics generation and wave steepening in a vacuum. Perhaps one of the simplest mechanisms. We choose the initial electromagnetic wave as

$$\bar{u}_0 = \bar{a}_1 \cos(kx)$$

and find from (**) that $\bar{u}(x,t) = \bar{a}_1 \cos\left(k(x - v_W t\right) - \frac{1}{2}\bar{a}_1^2 k v_W t \sin\left(2k(x - v_W t\right) + \ldots$

When a wave approaches the wave breaking point, wave steepening is equivalent to harmonics generation with higher numbers leading to the gradient catastrophe, i.e., at the shock wave.

The e.m. shock wave separates two regions, I and II, where the function $u(x,t)$ takes the values u_{II} and u_{II}. The shock wave front (it is an interface between regions I and II) moves with a velocity equal to u_{SW}, i.e. shock wave front is localized at $x_{SW} = v_{SW} t$. Integrating Eq. (*) over an infinitely small interval $(x_{SW} - \delta, x_{SW} - \delta)$, where $\delta \to 0$, we obtain

$$\left\{-v_{SW} u + v_W u - (4\varepsilon_2^2 + 3\varepsilon_3) W^5 u^2\right\}_{x = x_{sw}} = 0$$

where $\{f\}_x = f(x+\delta) - f(x-\delta)$ at $\delta \to 0$ denotes the discontinuity of function $f(x)$ at point x. This yields for the shock wave front velocity

$$v_{SW} = v_W - (4\varepsilon_2^2 + 3\varepsilon_3) W^5 (u_I + u_{II})$$

Shock wave front width should be of the order of the Compton wavelength, $\lambda\!\!\!^{-}_C = \hbar / m_e c$.

L' Oreal – UNESCO award for women in science 2019, awarded to Hedvika Kadlecová

H. Kadlecová, et al.,
Electromagnetic shocks in the quantum vacuum
Phys. Rev. D 99, 036002 (2019)

"Hedvika Kadlecová who works in the ELI Beamlines Research Center in Dolní Břežany became one of the three winners of the prestigious award L'Oréal-UNESCO for women in science for her theoretical work on nonlinear electromagnetic waves in quantum vacuum"

https://www.eli-beams.eu/en/media-en/hedvika-kadlecova-was-awarded-loreal-unesco-prize-for-women-in-science/

EUROPEAN UNION
European Structural and Investing Funds
Operational Programme Research, Development and Education
MINISTRY OF EDUCATION YOUTH AND SPORTS

Hodograph Transform

F.Pegoraro, SVB, Phys. Rev. D 100, 036004 (2019)

Nonlinear equations for electromagnetic wave in 1D case can be written as

$$\partial_+ w - \partial_- u = 0$$

$$\left[1 - 4\varepsilon_2 uw - 9\varepsilon_3 u^2 w^2\right]\partial_+ u - \left[\varepsilon_2 w^2 + 3\varepsilon_3 w^3 u\right]\partial_- u = \left[\varepsilon_2 u^2 + 3\varepsilon_3 u^3 w\right]\partial_+ w$$

with $\varepsilon_2 = (2/45\pi)\alpha$ and $\varepsilon_3 = (32/315\pi)\alpha$

System is linear with respect to highest order terms in partial derivatives ∂_+ and ∂_- with coefficients nonlinearly dependent on u and w, admits hodograph transformation. We consider x_- and x_+ as functions of u and w. This yields

$$\partial_{\mathrm{u}} x_+ - \partial_{\mathrm{w}} x_- = 0$$

$$\left[1 - 4\varepsilon_2 uw - 9\varepsilon_3 u^2 w^2\right]\partial_{\mathrm{w}} x_- + \left[\varepsilon_2 w^2 + 3\varepsilon_3 w^3 u\right]\partial_{\mathrm{w}} x_+ + \left[\varepsilon_2 u^2 + 3\varepsilon_3 u^3 w\right]\partial_{\mathrm{u}} x_+ = 0$$

It possesses reach family of particular solutions including Lorentz invariant solutions:

$$x^+ = \frac{C}{u\left(1 - 2\varepsilon_2 uw - 3\varepsilon_3 u^2 w^2\right)} \text{ and } x^- = \frac{C}{w\left(1 - 2\varepsilon_2 uw - 3\varepsilon_3 u^2 w^2\right)}$$

Discussions

Experiment : Measuring the phase difference between the phase of the electromagnetic pulse colliding with the counterpropagating wave and the phase of the pulse which does not interact with high intensity wave, it is equal to

$$\delta\psi = 4\pi \frac{d}{\lambda} \varepsilon_2 W^2$$

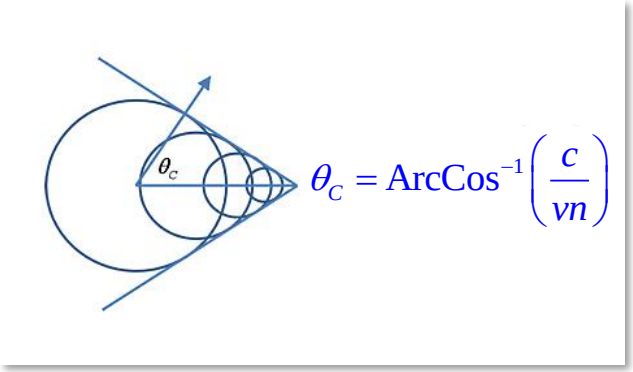
where λ is the wavelength of high frequency pulse and d is the interaction length, plays a central role in discussion of experimental verification of the QED vacuum birefringence.

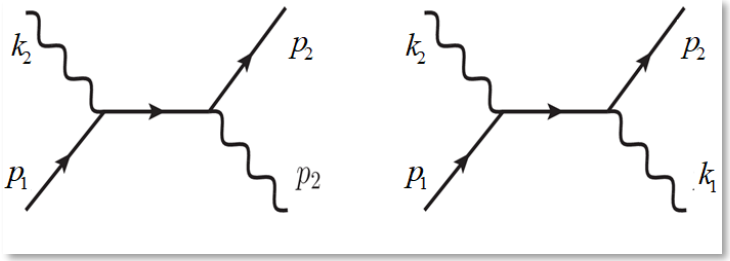
B. King and T. Heinzl, High Power Laser Sci. Eng. 4, 1 (2016); T. Heinzl, et al., Opt. Express 267, 318 (2006).

For 10 PW laser the radiation intensity can reach 10^{24} W/cm^2, for which $W^2 = 10^{-5}$. Taking the ratio d/λ equal to 10^4, i.e. equal to the ratio between the optical and x-ray radiation wavelength, we find that $\delta\psi = 10^{-4}$.

HOHG : Assuming $kv_W t = 2\pi d/\lambda$ and the intensity of the x-ray pulse of 10^{21} W/cm^2, we obtain that the ratio of the second harmonic amplitude to the amplitude of fundamental harmonic is approximately equal to 10^{-11}.

Synergic Cherenkov Radiation – Compton Scattering





Synergic Cherenkov Radiation - Compton Scattering

T. Erber, Rev. Mod. Phys. Rev. Mod. Phys. 38, 626 (1966); V. I. Ritus, Sov. Phys. JETP, 30, 1181 (1970); J. Schwinger, W. Y. Tsai, and T. Erber, Ann. Phys. 96, 303 (1976); W. Bekker, Physica 87A, 601 (1977); V. L. Ginzburg and V. N. Tsytovich, Phys. Reports 49, 1 (1979); I.M. Dremin, JETP Lett. 76, 151 (2002); S. V. Bulanov, P. V. Sasorov, S. S. Bulanov, G. Korn, Phys. Rev. D 100, 016012 (2019)

At the focus of 10 PW laser the field intensity can reach 10^{24}W/cm^2, i.e. $a_0 = 10^3$

Vacuum polarization changes the refraction index: radiation correction to the "photon mass" results in the dispersion equation for the e.m. wave frequency and wave vector

$$\omega^2 - k^2c^2 - \mu_{\|,\perp}^2 \frac{c^2}{\hbar^2} = 0$$

where (Ritus 1970)

$$\mu_{\|,\perp}^2 = -\alpha m_e^2 \begin{cases} \left[\frac{11\pm 3}{90\pi}\chi_\gamma^2 + i\sqrt{\frac{3}{2}}\frac{3\pm 1}{16}\chi_\gamma \exp\left(-\frac{8}{3\chi_\gamma}\right)\right] & \text{for } \chi_\gamma \ll 1 \\ \left[\frac{5\pm 1}{28\pi^2}\sqrt{3}\Gamma^4\left(\frac{2}{3}\right)(1-i\sqrt{3})\left(3\,\chi_\gamma\right)^{2/3}\right] & \text{for } \chi_\gamma \gg 1 \end{cases}$$

When $\alpha\chi_\gamma^{2/3} \to 1$ the "photon mass" tends to m_e.

In the limit $\chi_\gamma \ll 1$ the difference between the vacuum refraction index and unity is

$$\Delta n_\pm = \alpha \frac{11\pm 3}{45\pi}\left(\frac{E}{E_S}\right)^2$$

with $E_S = m_e^2c^3/e\hbar$ i. e. the normalized phase velocity of the e.m. wave equals

$$\beta_\pm = 1 - \varepsilon_\pm\left(\frac{E}{E_S}\right)^2$$

where $\varepsilon_\pm = \alpha(11\pm 3)/45\pi \approx 10^{-4}$.

Kinematics of SCRCS

Following to **V. L. Ginzburg (1940)** we describe the kinematics of electron-photon interaction in QED vacuum as γ

$$\boldsymbol{p}_0 + \hbar \boldsymbol{k}_0 = \boldsymbol{p} + \hbar \boldsymbol{k}, \quad m_e c^2 \gamma_0 + \hbar \omega_{\gamma,0} = m_e c^2 \gamma + \hbar \omega_\gamma$$

with $\gamma_0 = \sqrt{1 + p_0^2 / m_e^2 c^2}$, $\gamma = \sqrt{1 + p^2 / m_e^2 c^2}$, $\boldsymbol{p} = \mathrm{p}_{\|} \boldsymbol{e}_x + \mathrm{p}_{\perp} \boldsymbol{e}_y$, and $\boldsymbol{k} = |\boldsymbol{k}| (\cos\theta \boldsymbol{e}_x + \sin\theta \boldsymbol{e}_y)$

In medium with $n \neq 1$ the wave frequency ω and wave vector $\boldsymbol{k}$ are related to each other as $\boldsymbol{k} = \frac{\boldsymbol{k}}{|\boldsymbol{k}| c} n_{\pm} \omega$.

The photon energy is

$$\hbar \omega_\gamma = g \pm \sqrt{g^2 + 2 s \hbar \omega_0 \left(\frac{m_e c^2 \gamma_0 + p_{\|,0} c}{n_{\pm}^2 - 1} \right)}$$

where s is the number of photons and

$$g = \frac{\left(p_{\|,0} c - s \hbar \omega_0 \right) n_{\pm} \cos\theta - m_e c^2 \gamma_0 - s \hbar \omega_0}{n_{\pm}^2 - 1}$$

When the function g is positive and $s \hbar \omega_0 \ll m_e c^2 / \gamma_0$ the photon energy can be found to be

$$\hbar \omega_{Ch} \approx 2g + s \hbar \omega_0 \left(\frac{m_e c^2 \gamma_0 + p_{\|,0} c}{2(n_{\pm}^2 - 1)} \right)$$

This corresponds to the Cherenkov radiation.

In the opposite limit, when $g < 0$, in the limit $s \hbar \omega_0 \ll m_e c^2 / \gamma_0$ the Compton scattering mode frequency is given by

$$\hbar \omega_C \approx \frac{s \hbar \omega_0 \left(m_e c^2 \gamma_0 + p_{\|,0} c \right)}{\left(p_{\|,0} c - s \hbar \omega_0 \right) n_{\pm} \cos\theta - m_e c^2 \gamma_0 - s \hbar \omega_0}$$

In the limit In the limit $s < s_m$, where $s_m, = m_e c^2 / 4 \hbar \omega_0 \gamma_0$ the photon energy equals $\hbar \omega_C = 4 s \hbar \omega_0 \gamma_0^2$; at $s > s_m$, we have $\hbar \omega_C = m_e c^2 \gamma_0^2$.

Cherenkov radiation

Condition of Cherenkov radiation,

$$s\hbar\omega_0 \ll \frac{\sqrt{m_e^2c^4 + p_{\|,0}^2c^2} - p_{\|,0}cn_\pm\cos\theta}{1+n_\pm\cos\theta}$$

The electron energy should be large enough to have

$$\gamma_0 > \gamma_{Ch} = \frac{1}{\sqrt{2\Delta n_\pm}} = \sqrt{\frac{45\pi E_S^2}{\alpha(11\pm 3)E_0^2}} \approx 30\sqrt{\frac{I_S}{I_0}}$$

Here the laser intensity $I_0 = cE_0^2/4\pi$ in the focus region of 10 PW laser is approximately equal to 10^{24} W/cm^2;

$I_S = cE_S^2/4\pi \approx 10^{29}$ W/cm^2, i. e. the Cherenkov radiation threshold is exceeded for the electron energy above 10 GeV.

The Cherenkov cone with the angle $\theta_{Ch} = 2\sqrt{\varepsilon_\pm I_0/I_S}$ in the focus of 10 PW laser it is approximately equal to 2×10^{-5}.

The rate of the energy loss due to the Cherenkov radiation friction force along the electron trajectory is

$$\frac{d\mathcal{E}_e}{dx} = -\frac{e^2}{c^2}\int_{v_e n/c>1}\left(1-\frac{c}{v_e n_\pm}\right)\omega d\omega \approx -\frac{e^2}{\lambdabar_C^2}\varepsilon_\pm\left(\frac{E_0}{E_S}\right)^2$$

Integration is done over the region where $v_e n_\pm/c>1$.

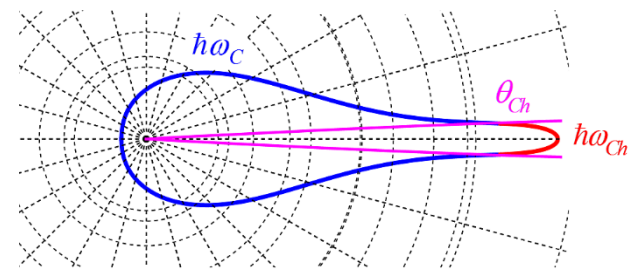
The formation length is given by

$$l_{Ch} \approx \lambdabar_C\gamma_e \quad (\lambdabar_C = \hbar/m_e c = 3.8\times10^{-11}\text{cm})$$

It is approximately equal to 2×10^{-5}.

Traversing the laser focus region the electron emits 0.2 photons.

For the electric charge of the LWFA electron bunch of 100 pC we obtain 10^4 photons.



Angle distribution of the energy logarithm for photon radiated by the SCCRS mechanism. Blue color used for Compton scattering and red color for the Cherenkov radiation. Magenta lines show the Cherenkov cone.

I. E. Tamm and I. M. Frank, Comptes Rendus de l'Accademie des Sciences de l'USSR 14, 109 (1937)

Probing the QED vacuum

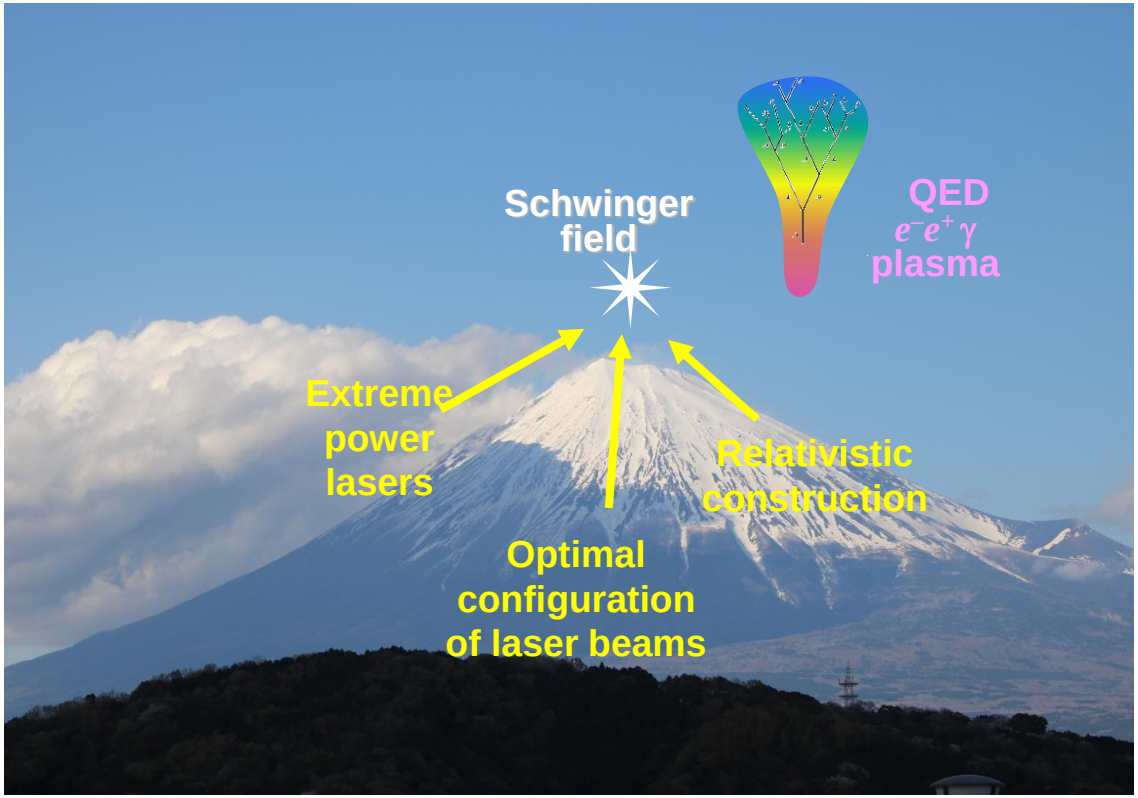
As one may see from the expression for the photon invariant mass

$$\mu_{\parallel,\perp}^2 = -\alpha m_e^2 \begin{cases} \left[\frac{11 \pm 3}{90\pi} \chi_\gamma^2 + i\sqrt{\frac{3}{2}} \frac{3 \pm 1}{16} \chi_\gamma \exp\left(-\frac{8}{3\chi_\gamma} \right) \right] & \text{for } \chi_\gamma \ll 1 \\ \left[\frac{5 \pm 1}{28\pi^2} \sqrt{3}\Gamma^4 \left(\frac{2}{3} \right) (1 - i\sqrt{3}) \left(3\,\chi_\gamma \right)^{2/3} \right] & \text{for } \chi_\gamma \gg 1 \end{cases}$$

at the high photon energy end, when the parameter χ_γ becomes larger than unity, the vacuum polarization effects weaken. In this limit the Cherenkov radiation does not occur.

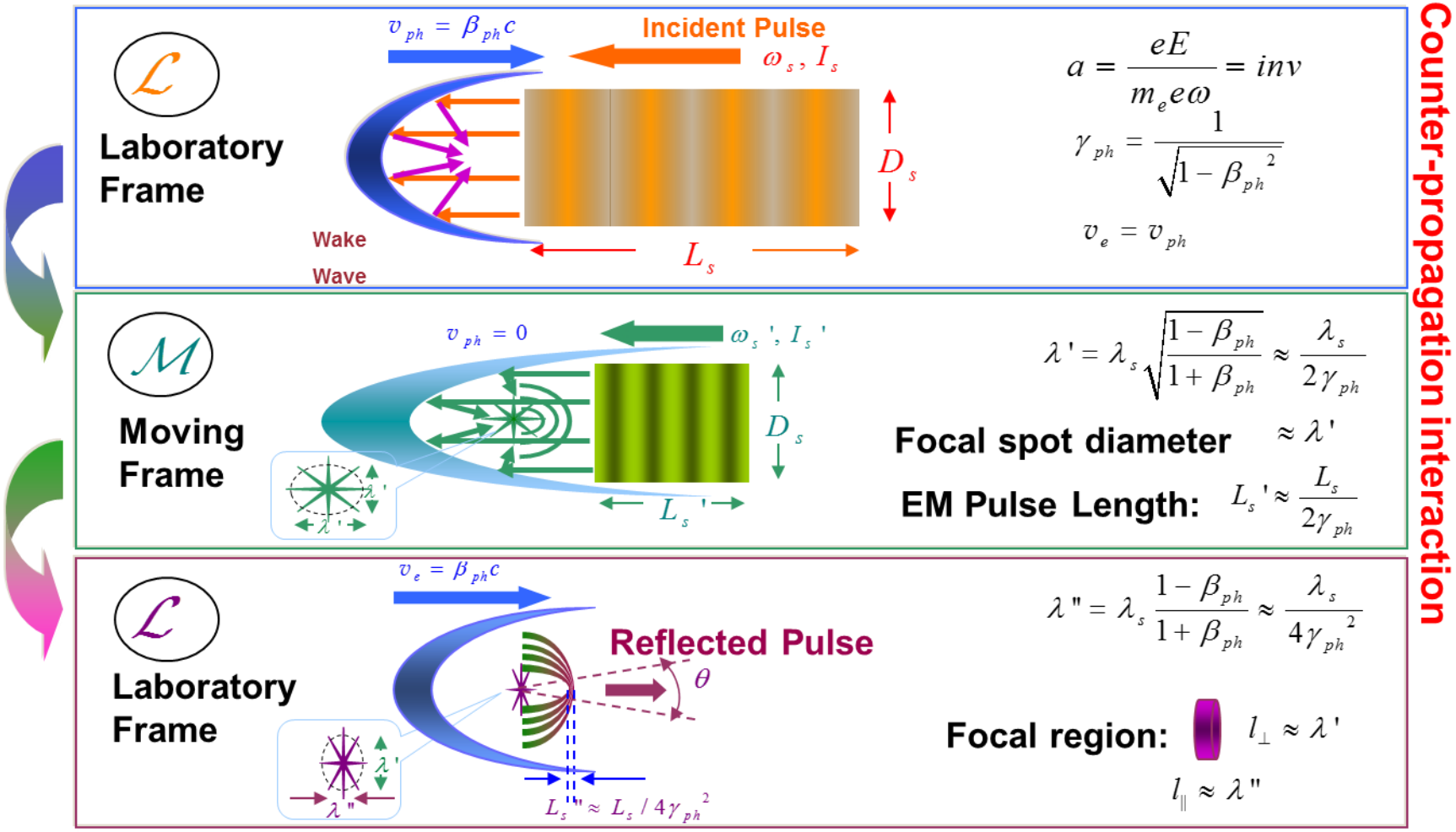
As a result the photons with the energy above $\hbar\omega_\gamma = m_e c^2 E_S / E_0$ are not present in the high frequency spectrum of the radiation. For 10 PW laser parameters this energy is approximately equal to 100 MeV.

Towards Schwinger Limit



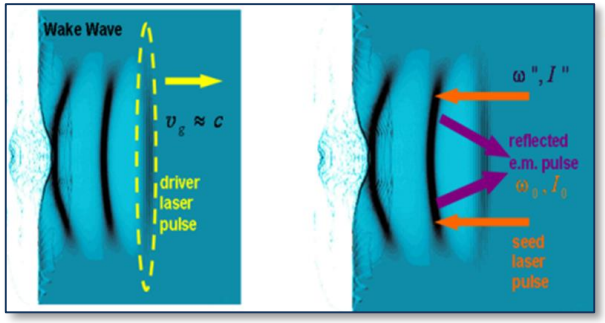
$$I_S = 10^{29}\,\frac{W}{cm^2} \quad \Rightarrow \quad \mathcal{P} = 10^{21}\left(\frac{\lambda}{1\mu m}\right)^2 W \quad \Rightarrow \quad \mathcal{E} = 3\times 10^6\left(\frac{\lambda}{1\mu m}\right)^3 J$$

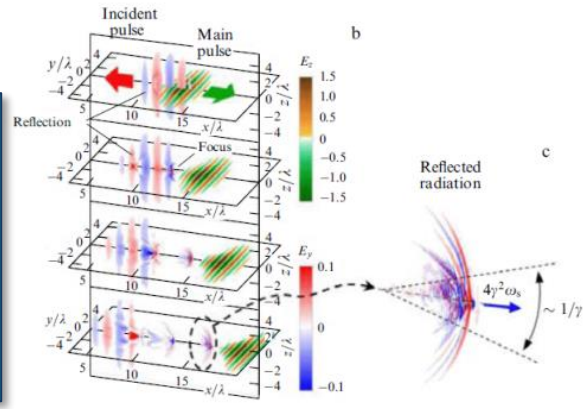
Relativistic Flying Mirror Concept



Relativistic Flying Mirror: Concept & Proof-of-Principle

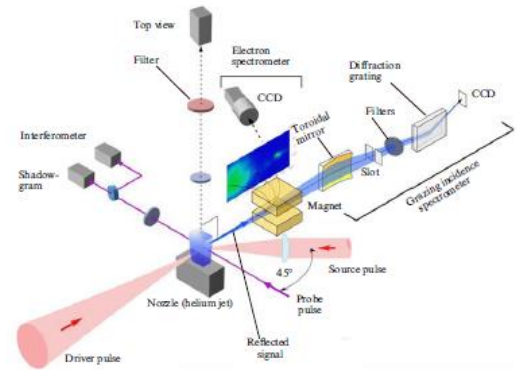
Concept:



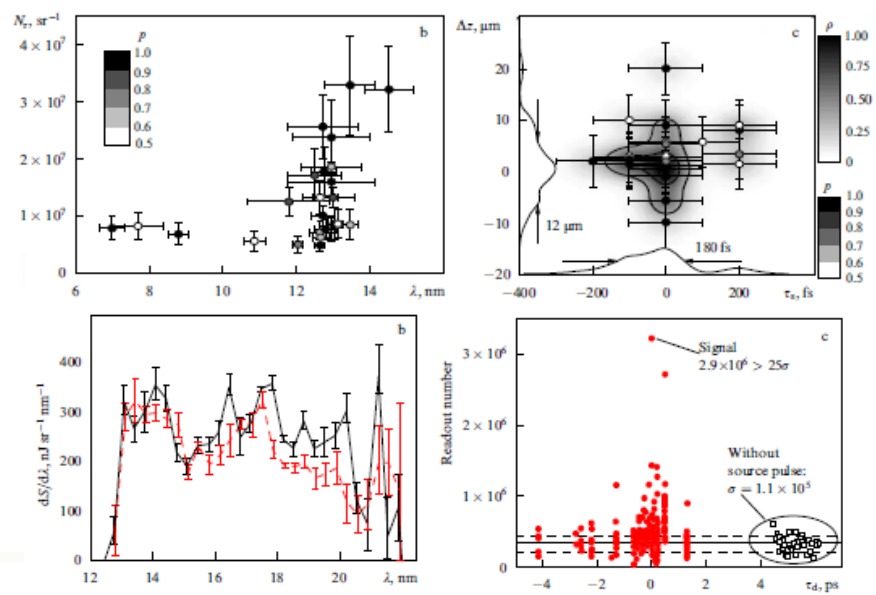


$$\omega_r = \omega_0 \frac{1+\beta_M}{1-\beta_M}, \quad I_r = I_0 \gamma_{ph}^6 \mathbf{R}\left(\frac{D}{\lambda_0}\right)^2$$

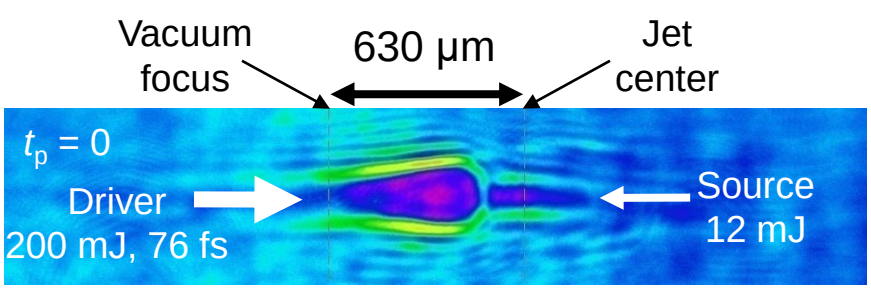
Experiments



Results



Space-Time Overlapping of Driver and Source Pulses



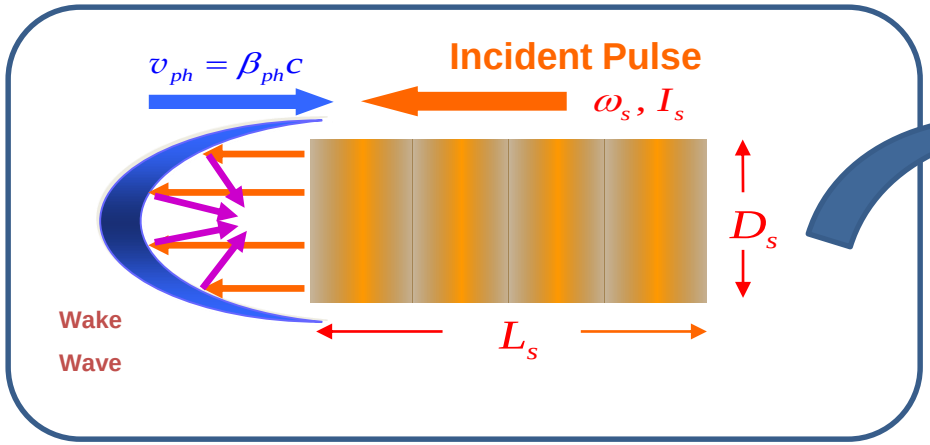
SVB et al., PRL 91, 085001 (2003); M.Kando et al., PRL 99, 135001 (2007); PRL 103, 235003 (2009); A. Pirozhkov, PoP 14, 123106 (2007); …… ; SVB et al., Phys. Uspekhi 56, 429 (2013), PSST (2016)

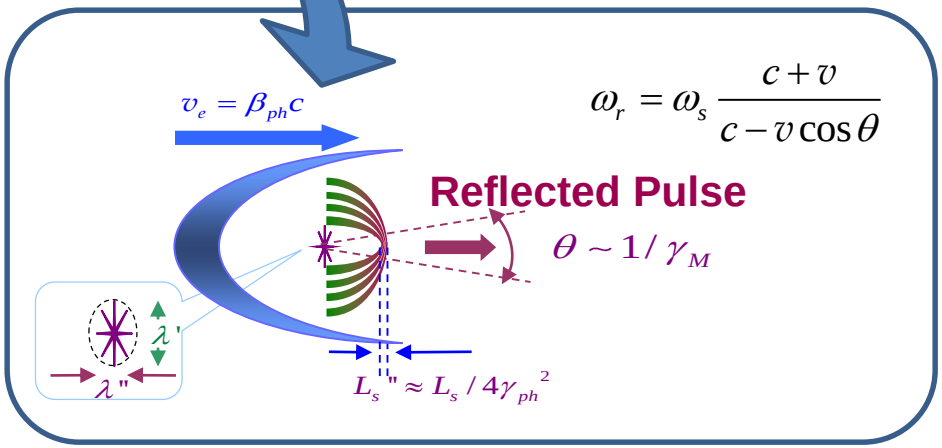
Towards Schwinger field ... and beyond

In a plane EM wave, both the invariants $\mathfrak{F} = F^{\mu\nu}F_{\mu\nu}$ & $\mathfrak{G} = F^{\mu\nu}\tilde{F}_{\mu\nu}$ **are zero,**

therefore e^-e^+ pairs are not created for an arbitrary magnitude of the EM field.

Relativistic flying mirror can create field γ_M times greater than QED critical field.





Laser Energy & Power to Achieve the Schwinger Field

The driver and source must carry 10 kJ and 30 J, respectively

Reflected intensity can approach the Schwinger limit

$$I_{QED} = 10^{29}\,\mathrm{W/cm^2}$$

It becomes possible to investigate such the fundamental problems of nowadays physics, as e.g. the electron-positron pair creation in vacuum and the photon-photon scattering

$$\mathcal{P}_{cr} = \frac{45\pi^2}{\alpha}\frac{cE_S^2\lambda^2}{4\pi}$$

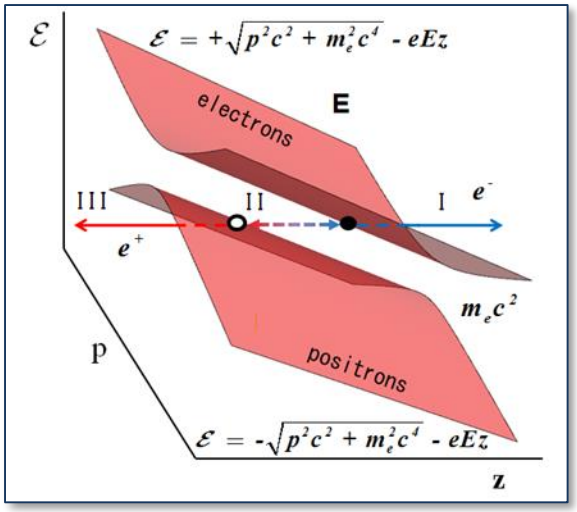
The critical power for nonlinear vacuum effects is $\mathcal{P}_{cr} \approx 2.5\times 10^{24}\,W$

Light compression and focusing with the FLYING MIRRORS yields $\mathcal{P} = \mathcal{P}_0\gamma_{ph}$

for $\lambda_0 = 1\mu m$ $\lambda = \lambda_0 / 4\gamma_{ph}^2$ **with** $\gamma_{ph} \approx 30$ **the driver power** $\mathcal{P}_{cr} = 100PW$

J. K. Koga et al., Phys. Rev. A 86, 053823 (2012)

Laser Driven $e^-e^+\gamma$ Plasma



e^- - e^+ Plasma in Terrestrial Laboratories

- Sub-MeV e^+ from radioactive isotopes or accelerators are used in material science:
A. P. Mills, Science 218, 335 (1982); P. J. Schultz and K. G. Lynn, Rev. Mod. Phys. 60, 701 (1988); A.W. Hunt et al., Nature (London) 402, 157 (1999); D.W. Gidley, et al., Ann. Rev. Mater. Res. 36, 49 (2006); B. Oberdorfer, et al., Phys. Rev. Lett. 105, 146101 (2010)

- e^+ emission tomography:
M. E. Raichle, Nature (London) 317, 574 (1985)

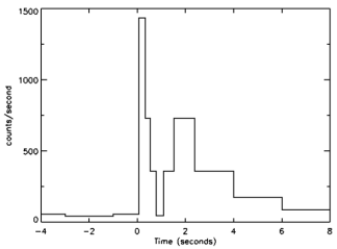
- Basic antimatter science such as antihydrogen experiments:
M. Amoretti, et al., Nature (London) 419, 456 (2002);
G. Gabrielse et al., Phys. Rev. Lett. 89, 213401 (2002)

- Bose-Einstein condensed positronium:
P. M. Platzman and A. P. Mills, Phys. Rev. B 49, 454 (1994)

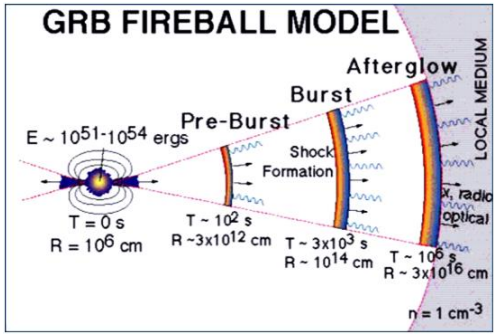
- Basic plasma physics:
C. M. Surko and R. G. Greaves, Phys. Plasmas 11, 2333 (2004)

e^-, e^+ Plasma in Space: Gamma Ray Bursts & the Leptonic Era of the Universe

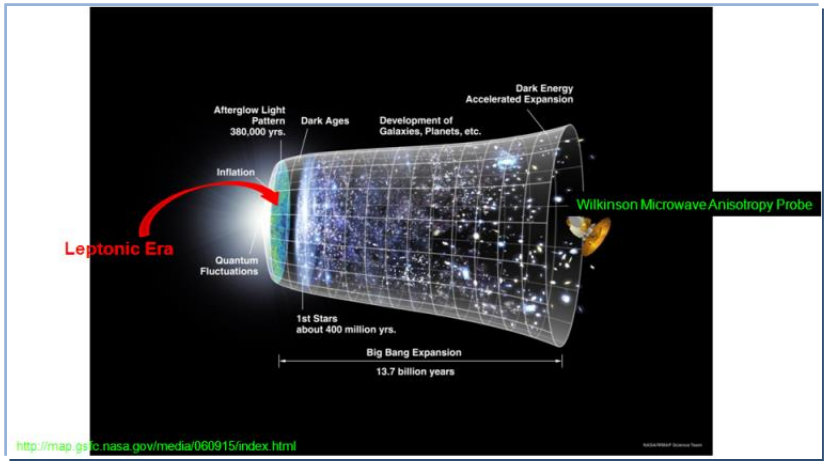
$E_{GRB} \approx 10^{53}$ erg

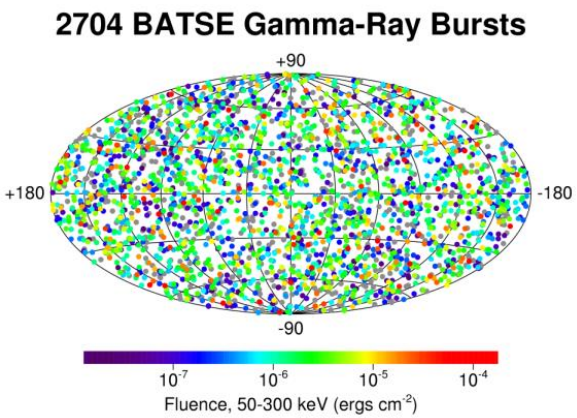


First GRB detected by Vela, 1967



(e^-, e^+, p, γ) - plasma





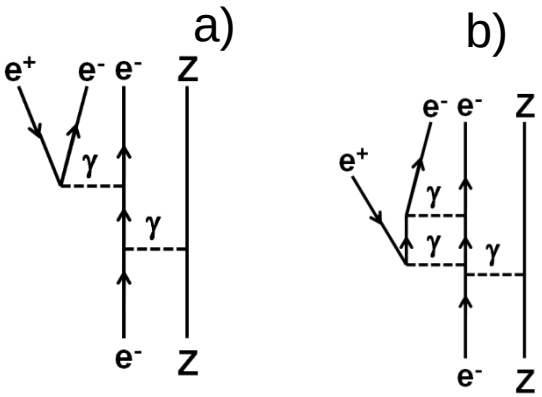
In the **Leptonic Era** the Universe consisted mainly of leptons and photons. It began when the temperature dropped below 10^{12} K some 10^{-4} seconds after the Big Bang, and it lasted until the temperature fell below 10^{10} K, at an era of about 1 second.

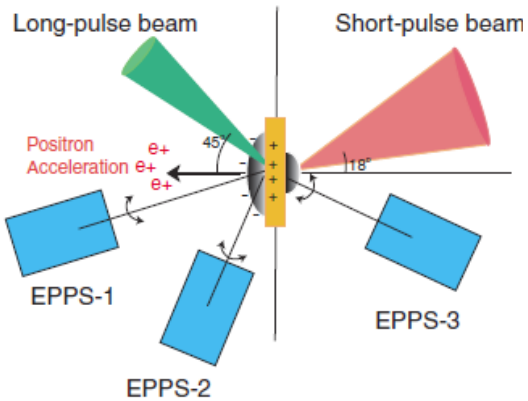
J. Silk, 'The Big Bang', New York: W. H. Publishers (1990)

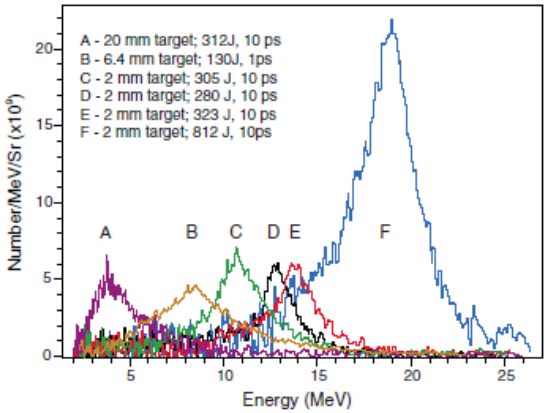
Meszaros, P., & Rees, M. J. 1992, MNRAS, 257, 29
Meszaros, P., & Rees, M. J. 1993, ApJ, 405, 278

e^- - e^+ pairs via trident + Bethe-Heitler processes

- **e^- - e^+** generation in laser-solid target interaction (**trident** mechanism of the **e^- - e^+ pair** generation)







a) *trident*

$$e^- + Z \rightarrow 2e^- + e^+ + Z$$

b) *bremsstrahlung*

$$e^- + Z \rightarrow \gamma + e^- + Z$$

pair production

$$\gamma + Z \rightarrow e^- + e^- + Z$$

$$\sigma_{e^-Z \rightarrow e^+e^-e^-Z} = (14/27\pi) r_e^2 (\alpha Z)^2 \ln\left(1 + p^2/m_e^2c^2\right)$$

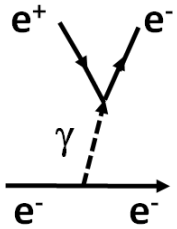
$$r_e = e^2/m_e c^2, \qquad \alpha = e^2/\hbar c = 1/137$$

C.Gahn, et al., Phys. Plasmas 9, 987 (2002)
H.Chen, et al., Phys. Plasmas 16, 122702 (2009)
H.Chen, et al., Phys. Rev. Lett. 105, 015003 (2010)

Breit-Wheeler process of e⁻ - e⁺ generation

Breit-Wheeler process of **e⁻ - e⁺** generation $\hbar\omega + \hbar\omega' \rightarrow e^+ + e^-$ with $\sigma_{\omega\omega\rightarrow e^+e^-} \approx \alpha^2 r_e^2$

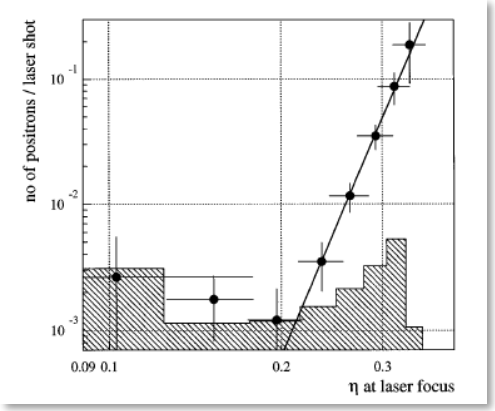
if $\hbar^2\omega\omega' > m_e^2 c^4$

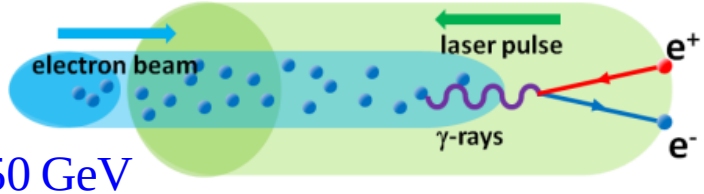


Multiphoton inverse Compton $e^- + N\hbar\omega_0 + \rightarrow \hbar\omega_\gamma + e^-$

&

Multiphoton B-W processes $\hbar\omega_\gamma + N\hbar\omega_0 \rightarrow e^+ + e^-$





50 GeV

$1.6\,ps \quad a_0 \approx 0.3$

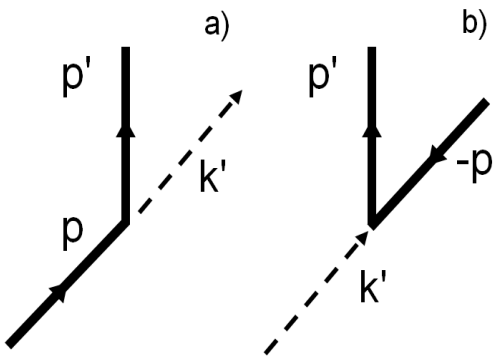
$$\chi_e = (E/E_S)\gamma_e \approx 0.3$$

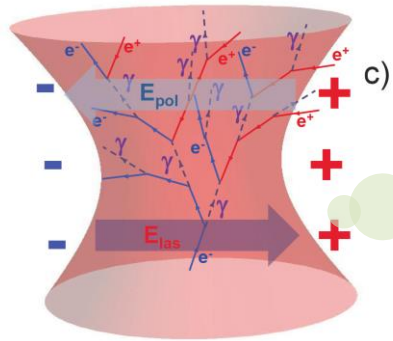
$$\chi_\gamma = (E/E_S)(\hbar\omega_\gamma / m_e c^2) \approx 0.15$$

G. Breit, J. A. Wheeler, Phys. Rev. 46, 1087 (1934)
D. L. Burke, et al., Phys. Rev. Lett. 79, 1627 (1997)

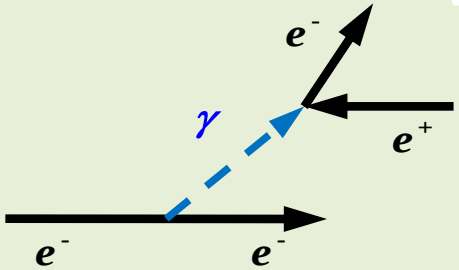
Electromagnetic cascades induced by pairs

$$\chi_e = \frac{e\hbar\sqrt{\left(F_{\mu\nu}p_\mu\right)^2}}{m_e^3c^4} = \gamma_e\frac{E}{E_S} \gg 1 \quad \rightarrow \quad n_+ \approx n_- = \frac{E}{4\pi e\lambda_0}$$





$$I > 2.5\times10^{24} W/cm^2$$



D.L.Burke et al., PRL 79, 1626 (1997)

$$\chi_e \approx 0.3, \quad \chi_\gamma \approx 0.15$$

•W.Heisenberg, H.Euler (1936)
•N. B. Narozhny, "Extreme Power Photonics: New Phenomena" ISTC-ELI/HiPER/PETAL Workshop, 2008
•A. R. Bell and J. G. Kirk, "Possibility of Prolific Pair Production with High-Power Lasers " Phys. Rev. Lett. 101, 200403 (2008)
•A. M. Fedotov, N. B. Narozhny, G. Mourou, G. Korn, "Limitations on the Attainable Intensity of High Power Lasers" PRL 105, 080402 (2010)

Probability of pair creation

$$W(\chi_\gamma) = \frac{3}{32}\frac{e^2 m_e^2 c^3}{\hbar^3 \omega_\gamma}\left(\frac{\chi_\gamma}{2\pi}\right)^{3/2} \exp\left(-\frac{8}{3\chi_\gamma}\right) \quad \text{for } \chi_\gamma \ll 1$$

$$W(\chi_\gamma) = \frac{27\Gamma^7(2/3)}{56\pi^5}\frac{e^2 m_e^2 c^3}{\hbar^3 \omega_\gamma}\left(\frac{3\chi_\gamma}{2}\right)^{2/3} \quad \text{for } \chi_\gamma \gg 1$$

The number of absorbed laser photons:

$$N_l \gg a$$

Photon mean-free-path before the pair is created:

$$l_{m.f.p} = \frac{\lambda_0}{0.2\pi\alpha\, a} \approx 220\frac{\lambda_0}{a}$$

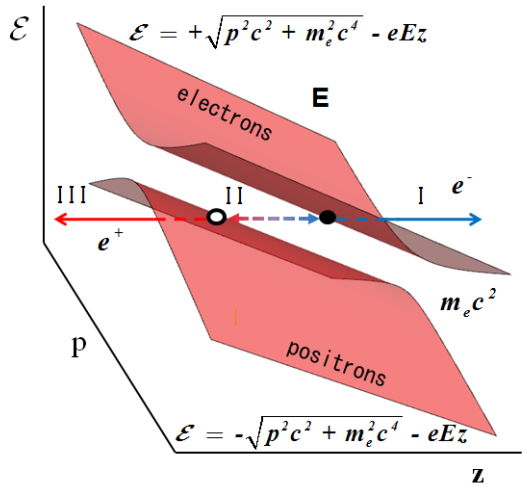
THRESHOLD: $\chi_\gamma > 1$ for $\chi_\gamma \approx \chi_e$ and $\gamma_e = a$

$$\chi_e = \frac{a}{a_S}\gamma_e \approx \frac{a^2}{a_S} > 1 \text{ gives } a > \sqrt{a_S} \approx 10^3 \text{ gives } I > 10^{24} W/cm^2$$

Here $a_S = eE_S / m_e \omega c = m_e c^2 / \hbar\omega$

A. M. Fedotov, N. B. Narozhny, G. Mourou, G. Korn, "Limitations on the Attainable Intensity of High Power Lasers" PRL 105, 080402 (2010)

“Schwinger” mechanism of e^- - e^+ pair creation



Energy is plotted against momentum and coordinate (electric field is parallel z - axis). Solution to Dirac equation shows that the ψ - function is large only in the regions I and III. In the region II, it decreases exponentially. Therefore transmission coefficient through region II, which gives probability of pair creation, has the order of magnitude $\exp(-\pi m_e^2c^3 / |e| \hbar E)$

Quasiclassical regime (F. Sauter, 1931)

$$w = A\exp\left(-\frac{2}{\hbar}\int_{z_1=-m_ec^2/eE}^{z_2=+m_ec^2/eE} |p(z)|\,dz\right)$$

$$= A\exp\left(-\frac{4m_e^2c^3}{|e|E\hbar}\int_0^1 \sqrt{1-\xi^2}\,d\xi\right)$$

$$\propto \exp\left(-\frac{\pi E_S}{E}\right)$$

Probability (W.Heisenberg, H.Euler, 1936)

$$w = \frac{1}{4\pi^3}\left(\frac{|e|E\hbar}{m_e^2c^3}\right)^2 \frac{m_ec^2}{\hbar}\left(\frac{m_ec}{\hbar}\right)^3 \exp\left(-\frac{\pi m_e^2c^3}{|e|\hbar E}\right)$$

Rate of Pair Generation

$$\mathfrak{F} = \frac{\boldsymbol{E}^2 - \boldsymbol{B}^2}{2} = \frac{F_{\mu\nu}{}^2}{4} = inv, \quad \mathfrak{G} = (\boldsymbol{E}\cdot\boldsymbol{B}) = \frac{F_{\mu\nu}F_{\mu\nu}{}^*}{4} = inv$$

$$\mathfrak{E} = \sqrt{\sqrt{\mathfrak{F}^2 + \mathfrak{G}^2} + \mathfrak{F}}, \quad \mathfrak{B} = \sqrt{\sqrt{\mathfrak{F}^2 + \mathfrak{G}^2} - \mathfrak{F}}$$

are the electric and magnetic fields in the frame where they are parallel

$$W_{e^+e^-} = \frac{e^2 E_S^2}{4\pi^3\hbar^3 c}\mathfrak{a}\mathfrak{b}\coth\left(\frac{\pi\mathfrak{b}}{\mathfrak{a}}\right)\exp\left(-\frac{\pi}{\mathfrak{a}}\right)$$

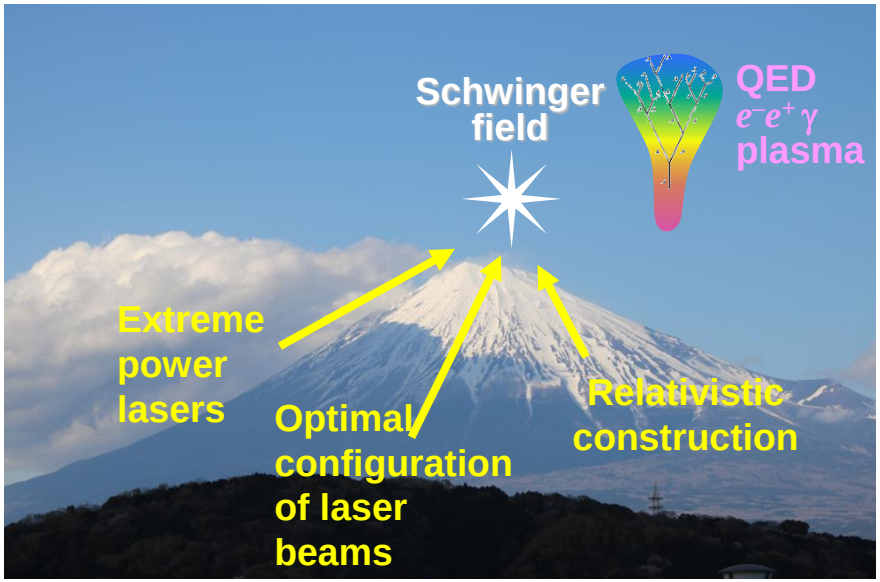
$$\mathfrak{b} = \mathfrak{B}/E_S, \quad \mathfrak{a} = \mathfrak{E}/E_S$$

for $\mathfrak{e} \to 0$ the rate $W_{e^+e^-} \to 0$

at $\mathfrak{b} \to 0$ the rate $\quad W_{e^+e^-} = \dfrac{e^2 E_{Schw}^2}{4\pi^2\hbar^3 c}\mathfrak{a}^2\exp\left(-\dfrac{\pi}{\mathfrak{a}}\right)$

The number of e^+e^- pairs $\quad N_{e^+e^-} = \iint W_{e^+e^-}\,dVdt = \dfrac{c\tau l_x l_y l_z}{64\pi^4\lambda\!\!\!^{-}_C{}^4}\mathfrak{a}_m^4\exp\left(-\dfrac{\pi}{\mathfrak{a}_m}\right)$

Multiple Laser Beam Configuration

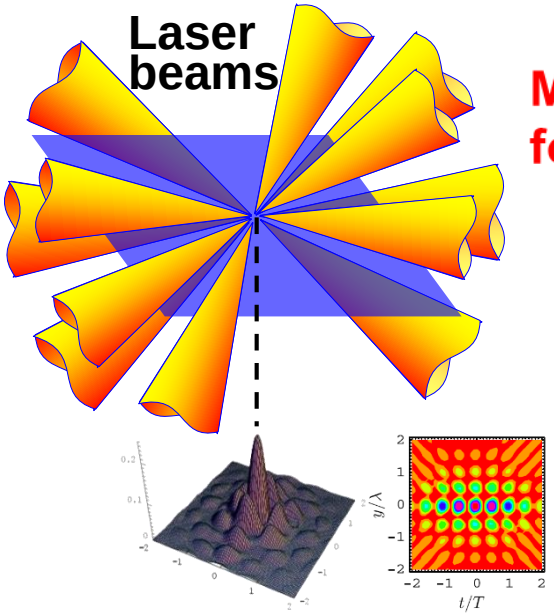


Electron-positron pairs can be created before the laser field reaches the Schwinger limit, due to a large phase volume occupied by a high-intensity EM field.

S. S. Bulanov, N. B. Narozhny, V. D. Mur, V.S. Popov, "Electron-positron pair production by electromagnetic pulses". JETP, 102, 9 (2006)

A. R. Bell & J. G. Kirk, "Possibility of Prolific Pair Production with High-Power Lasers". Phys. Rev. Lett. 101, 200403 (2008)

A. M. Fedotov, N. B. Narozhny, G. Mourou, G. Korn, "Limitations on the Attainable Intensity of High Power Lasers". Phys. Rev. Lett. 105, 080402 (2010)

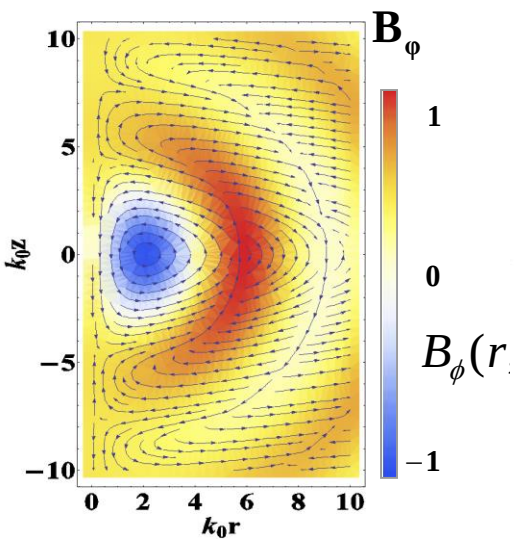


Multiple 10kJ beam system provides necessary conditions for e^-e^+ pairs creation.

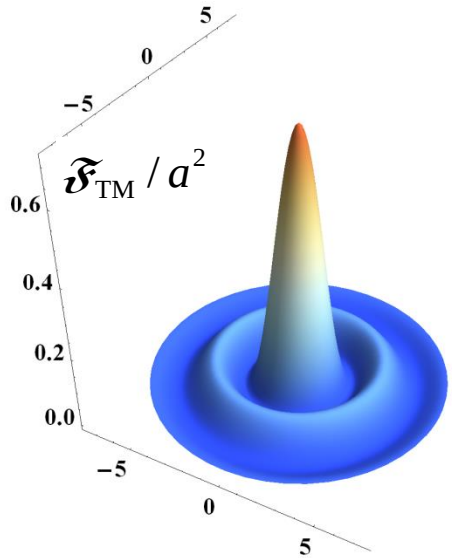
Number of pulses	Number of e^-e^+ with 10kJ pulses	Required energy (kJ) to create one pair
2	9×10^{-19}	40
4	3×10^{-9}	20
8	4	10
16	1.8×10^{3}	8
24	4.2×10^{6}	5.1

S.S.Bulanov, V.D.Mur, N.B.Narozhny, J.Nees, V.S.Popov, Phys. Rev. Lett. 104, 220404 (2010)

3D EM configuration TM - mode



$\boldsymbol{E}(r,z)$



TM configuration :

Magnetic field

$$\boldsymbol{B}(R,\theta) = \\ = \boldsymbol{e}_\phi \frac{B_0 \sin(\omega_0 t)}{(8\pi k_0 R)^{1/2}} J_{n+1/2}(k_0 R) L_n^l(\cos\theta)$$

Electric field

$$\boldsymbol{E} = i k_0^{-1} (\nabla \times \boldsymbol{B})$$

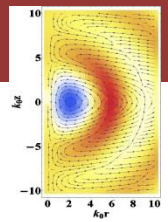
The vector field shows *r*- and *z*-components of the poloidal electric field in the plane *(r,z):*

The color density show toroidal magnetic field distribution.
The first Poincare invariant

$$\frac{\mathfrak{F}_{\mathrm{TM}}}{a_0^2} = \frac{\mathbf{E}^2 - \mathbf{B}^2}{2a_0^2}$$

S.S.Bulanov, T. Zh.Esirkepov, A.Thomas, J.Koga, S.V.Bulanov, "On the Schwinger limit attainability with extreme power lasers" Phys. Rev. Lett. 105, 220407 (2010)

Threshold for the e$^-$ - e$^+$ avalanche



INSTABILITY : e$^-$ (e$^+$)trajectory in r, z - plane $\omega_0 t = 1,\ a_0 \gg 1$

$$p_z(t) = m_e c\, a_0 \omega_0 t,$$

$$p_r(t) = m_e c \frac{a_0 k_0 r_0 \omega_0 t}{2^{3/2}} I_1\left(\frac{\omega_0 t}{2^{3/2}}\right),$$

$$r(t) = \frac{a_0 r_0}{2^{3/2}} I_1\left(\frac{\omega_0 t}{2^{3/2}}\right) + \frac{a_0 r_0 \omega_0 t}{16}\left[I_0\left(\frac{\omega_0 t}{2^{3/2}}\right) + I_2\left(\frac{\omega_0 t}{2^{3/2}}\right)\right]$$

with $k_0 r_0 = (2.5 a_0 / \pi a_S)^{1/2}$; growth rate : $\Gamma = \omega_0 / 2^{3/2}$

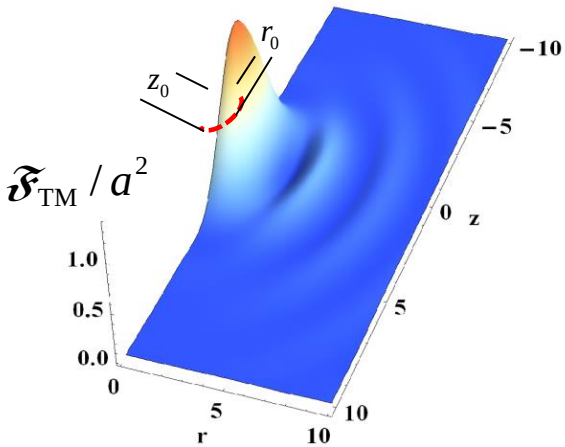
THRESHOLD : assuming $\omega_0 t \gg 0.1\pi$ we find that the avalanche will start at $a_0 / a_S \gg 0.105$

which corresponds to $I^* \gg 4 \times 10^{27}$ W/cm^2

The pairs are created in a small 4-volume near the electric field maximum (maximum of $\mathcal{F}_{\mathrm{TM}}$) with the characteristic size

$$\pi r_0^2 z_0 c t_0 = \frac{5^{3/2}\lambda_0^4}{16\pi^5}\left(\frac{a_0}{a_S}\right)^2$$

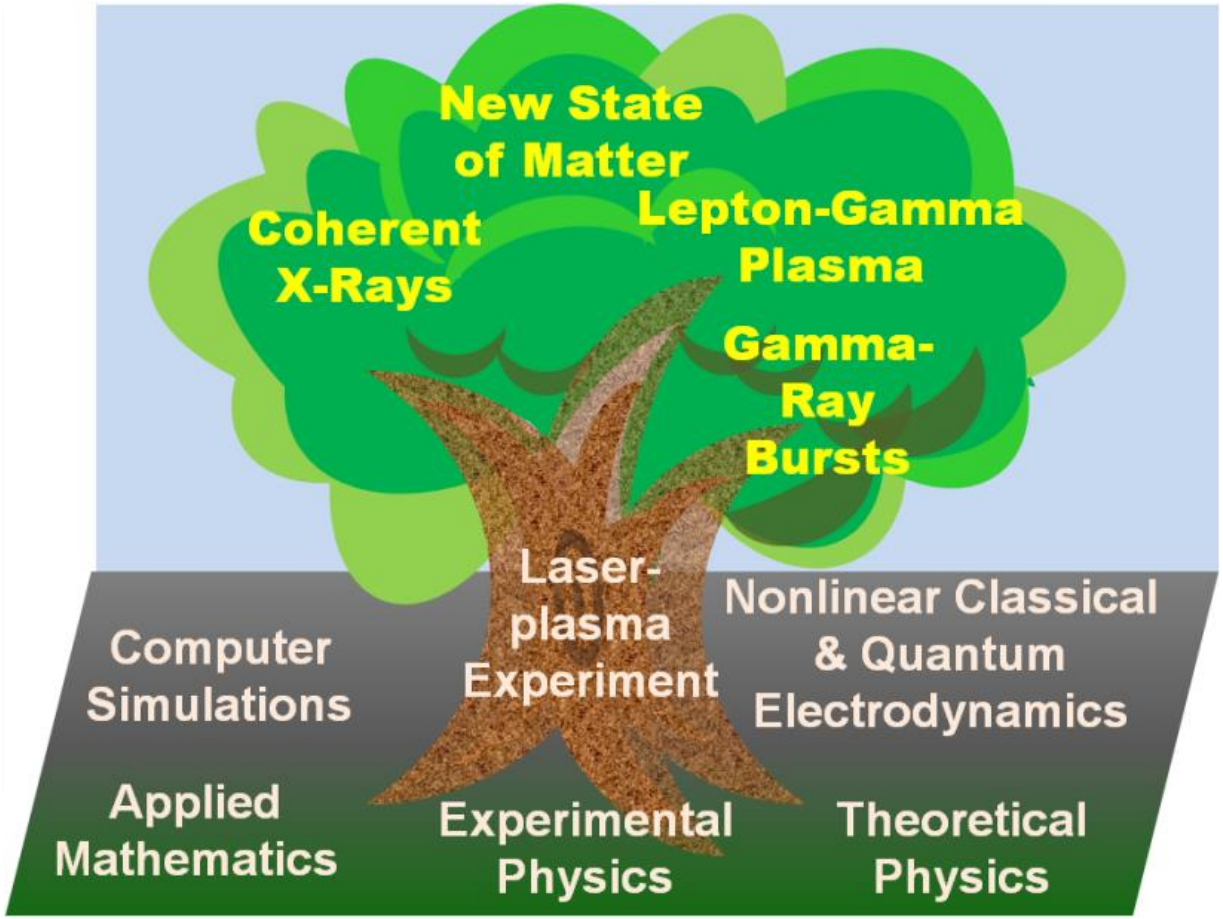
Here normalized Schwinger field is $a_S = \dfrac{m_e c^2}{\hbar\omega_0} = 5.1\times 10^5$



Integrating over the 4-volume the probability of the pair creation we obtain the number of pairs produced per wave period,

$$N_{e^+e^-} = \iint W_{e^+e^-}\, dV dt = \frac{5^{3/2}}{4\pi^3} a_0^4 \exp\left(-\pi\frac{a_S}{a_0}\right)$$

he first pairs can be observed for an one-micron wavelength laser intensity of the order of $\approx 10^{27}\,\mathrm{W/cm^2}$, which corresponds to $a_0 / a_S = 0.05$, i.e. a characteristic size, r_0 , is approximately equal to $0.04\,\lambda_0$



Thank you for listening to me!